Periodical waves in the evolution of art: methods of study

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Abstract

Studies of periodical processes in the evolution of art became rather widespread. P. Sorokin described cycles of about several centuries; periods close to 50 years have been observed in social relations by S. Maslov; numerous cycles in the stylistic evolution of art became well known due to C. Martindale. One of important characteristics of art history is the ‘intensity of artistic creativity’, which can be measured as the total (summary) volume of encyclopaedic descriptions devoted to artists of appropriate creative sphere during every temporal segment of 1..10 years. These rows of ‘experimental data’ form evolutionary curves on historical time interval of several centuries. Such curves contain two components: a long-term trend and an ‘oscillating part’, which have time constants of about decades of years. In the row of historical data, these oscillations may be represented with only 2..10 ‘waves’ including 2 to 10 sampling points pro oscillation period. The goal of investigation is, to measure parameters of such oscillating components.

Keywords: Art evolution, stylistic evolution, intensity of artistic creativity, periodical waves, trend evaluation, spectrum analysis

1. Periodicity studies of art evolution

Studies of periodical processes in the evolution of art style made by P. Sorokin discovered cycles of about several centuries [1]. In the theory of economical evolution [2], periods close to 50 years have been described. The same period have been found in changes of social ‘climate’ and architecture style [3]. Numerous evolution cycles of different kinds of art have been analyzed in [4]. Another important aspect for the artistic evolution is the intensity of artistic creativity measured as the number of events and/or its significance — see, e.g., [5, 6], or the number of creativity persons itself [7].

The evolution parameters to be analyzed will be fixed for each small temporal segment (e.g., each given 1-year, 5-year or 10-year interval). In case of stylistic studies, these parameters will be the expert’s estimations of artistic works (paintings, musical pieces, architecture etc.); for intensity analysis, the integrative parameter is the total (summary) volume of descriptions devoted to persons which were active in the creative sphere considered during the given temporal segment. These rows of ‘experimental data’ form evolutionary curves on time interval about 200-1000 years long.

As a rule, such curves contain two components: a long-range trend which is usually firstly growing, and then falling down, and some short-range waves (oscillations) against the background of this trend. The growing of long-range trend describes the actual growing of creative intensity in the studied branch of art. The hill-like form may appear because of the superposition of two subjective factors: 1) the decay of interest in composers (painters, dramatists’ etc.) of more and more remote eras and 2) the decreasing evaluation of contemporary composers (painters, etc.), because for the compilers of the encyclopedia, now it is still difficult to forecast which artists will become ‘classics’, and therefore deserve a wordy description. Due to first factor, the indicator of intensity for rather remote epochs
should show increasing with time; due to second factor, this indicator for the contemporary time diapason should decrease. The superposition of these two tendencies may result a hill-like behavior of the long-range trend. These hill-like evolution embraces time diapason of several centuries.

The main object of our interest will be exactly short-range waves which are observed against the background of the long-range trend. In order to analyze only the ‘oscillating part’ of evolutionary curve, firstly the long-term trend must be estimated and then extracted. One of problems is to separate, if possible, in any ‘optimal’ way, the trend-component from the oscillation-component.

2. Mathematical model

The ‘true’ trend curve is unknown; the trend itself denotes a kind of ‘mean value’, which varies slowly (during hundred of years) with the time. The oscillations to be studied have time constants of about decades of years. Decomposition of any evolutionary curve into two components, ‘trend part’ and ‘oscillating part,’ means presentation of this curve by model:

\[ Y(t) = F_{tr}(t) + F_{osc}(t), \]

where \( F_{tr}(t) \) is the trend function, and \( F_{osc}(t) \) is the ‘oscillating part.’

There exist various ways to calculate the trend. In our investigation we resorted to the help of polynomial approximation of each trend curve with the use of minimal mean square error criteria [6]. Thereby, the ‘trend part’ \( F_{tr}(t) \) will be described as a polynomial of degree ‘\( N \)’:

\[ F_{tr}(t) = b_0 + b_1 \times t + b_2 \times t^2 + \ldots + b_N \times t^N; \] \hspace{1cm} (2)

\( b_0, b_1, b_2, \ldots, b_N \) being constant coefficients.

The simplest trend form is a straight line (\( N = 1 \)). The next one is the quadratic parabola (\( N = 2 \)), a more complex trend line is the 3\(^{rd} \) power parabola and so on. As it is well-known, increasing of the polynomial power \( N \) will cause better and better coincidence of empirical and approximating curves and, therefore, decreasing of mean square error, \( \varepsilon(N) \).

The ‘oscillating part’ can be presented as a sum of trigonometric functions:

\[ F_{osc}(t) = a_1 \sin(\omega_1 t + \varphi_1) + a_2 \sin(\omega_2 t + \varphi_2) + \ldots + a_M \sin(\omega_M t + \varphi_M) + \eta(t) \] \hspace{1cm} (3)

where \( a_1, a_2, \ldots, a_M \) are the amplitudes of oscillating components, \( \omega_1, \omega_2, \ldots, \omega_M \) — their frequencies, \( \varphi_1, \varphi_2, \ldots, \varphi_M \) — phases of sinusoidal oscillations, and \( \eta(t) \) is a possible stochastic component.

Parameters \( a_1, a_2, \ldots, a_M \) relating to the frequency points \( \omega_1, \omega_2, \ldots, \omega_M \) are responsible for the spectrum of ‘oscillating part’ \( F_{osc}(t) \). In case of strong periodical function \( F_{osc}(t) \), spectrum will contain a set of components with multiple frequencies \( \omega_1, 2 \times \omega_1, 3 \times \omega_1, \ldots \) — it will be the well-known Fourier row. In this case, period is equal to \( T_p = 2\pi / \omega_1 \), where \( \omega_1 \) is the ‘fundamental’ frequency of oscillations. In more complex cases, \( F_{osc}(t) \) will contain many oscillating components with aliquant frequencies.

3. Analysis algorithm

The main goal of this investigation is to study the ‘oscillating part’, which should be detached from the ‘trend part’ of the empirical curve. In order to make the separation, firstly a polynomial approximation of the trend line with the given power \( N \) will be calculated (2), and then the trend curve will be subtracted from the empirical one (that is a ‘centering’ operation). For every given \( N \), a certain set of trend polynomial coefficients \( b_0, b_1, b_2, \ldots, b_N \) and an appropriate error value \( \varepsilon(N) \) will be calculated. The oscillating part \( F_{osc}(t) \), which is
equal to the difference $Y(t) - F_r(t)$, changes in dependence of $N$. Let’s note, that the parameters of its spectrum, $a_1, a_2, \ldots, a_M$ and $\omega_1, \omega_2, \ldots, \omega_M$, also depend from $N$.

We may expect that in a certain range of $N$, error values $\varepsilon(N)$ and the sets of spectrum parameters will become relatively stable. In this range, the polynomial power $N$ will be high enough to provide a precise approximation of the ‘true’ mean line of the evolutionary curve, but not high enough to represent high-speed changes of oscillating components. Our investigation of empirical data relating to the intensity of creativity in Russian and West European music, painting and theatre showed that this ‘stability interval’ does really exist for appropriate curves. For example, on Fig. 1, the evolutionary curves for Italian music ‘creativity intensity’ and the trend form for $N = 9$ is shown (time step: 10 years; 45 data points) [5].

![Figure 1](waves_ex_c_akherto.2004 source file: itml)

**Fig. 1.** Italian music ‘creativity intensity’: a trend in form of polynomial with $N = 9$.

Figure 2a presents the characteristic dependence of mean square approximation error $\varepsilon(N)$ for $N$ values from 1 to 12. On Fig. 2b, the tracks of spectral peak positions are shown in dependence of $N$. As it was shown above, the form of ‘oscillating part’ of evolutionary curve varies in dependence of $N$. Therefore, the spectrum of the curve will be changed also. For every value of ‘$N$’, positions for pronounced spectral peaks have been compiled, and Fig. 2b is the graphical representation of these frequency positions.

![Figure 2](a) Mean square approximation error; (b) Spectrum peak positions.

**Fig. 2.** Trend approximation (intensity of Italian music)

The study of $\varepsilon(N)$ on Fig. 2a shows that the increasing of $N$ after $N = 8..9$ does not cause significant variety of error value. Spectrum peak positions (see Fig. 2b) become stable after $N = 9$. In this case, we use as the trend model a polynomial of the $9^{th}$ power.

The next step of study is a more precise spectrum analysis of de-trended curves. For this operation, the possible frequency range is limited from above by Kotelnikov-Niquist.
frequency $F_{\text{max}} = 1/(2 \times \Delta t)$, where $\Delta t$ is the ‘time step’ — time interval (1, 5, 10… years) on which the experimental data have been summarized. The lower frequency limitation is $F_{\text{min}} = 1/(2 \times T_A)$, where ‘$T_A$’ is the whole time interval where the evolution have been analyzed: it must include at least 2 periods of oscillations in order to argue the periodicity.

In some cases, row data can contain components which do not conform the first of these conditions, i.e. some oscillations may be ‘too fast’ for the chosen time step. Appropriate low-frequency filtration can help avoiding the influence of higher frequencies. This task was solved by means of ‘smoothing’ of each curve in time domain with the use of a weight function. In our calculations, the triangle function have been used with the effective width (smoothing time) about $\tau = 55$ years.

The oscillation spectrum for creation intensity in Italian music (after de-trending and smoothing) is shown on Fig. 3.

![Fig. 3. Spectrum of ‘oscillating part’ for creation intensity (Italian music).](image)

Internal maximums of spectrum correspond to harmonical oscillation components. In statistics analysis, the estimation of component’s frequencies based on these maximums is known as ‘maximum likelihood’ method. Usually, the calculated spectrums have many such maximums, but some of them are ‘method artifacts’. The reason of this disparity is calculating of spectrums on a finite time interval $T_A$. In this case, every harmonical component with the oscillation frequency $f_0$ produces a spectrum function

$$S(f) = S_0 \frac{\sin \pi (f - f_0)T_A}{\pi (f - f_0)T_A},$$

where $S_0$ is an amplitude constant. The width of ‘main lobe’ of this function (between zeros) is $\Delta F = 2/T_A$. Two components with a frequency difference later then $\Delta F$ will not be recognized as independent harmonics. For more differentiate components, overlapping (spectral interference) of main lobes and side-lobes belonging to adjacent component of spectrums will deform spectrum maximums and shift them to higher or lower frequency.

In order to select only really existing components, a re-construction method can be suggested which uses re-generating of detected components with further optimizing of component parameters with the criteria of minimum mean square approximation error [6]. It means that a minimum of mean-square error will be found for a set of harmonics selected from the spectrum like shown on Fig. 3. For searching the minimum, both Monte-Carlo search method and multidimensional gradient descent method have been used. In case of simulated data rows which include a sum of 2-3 sinusoidal components, the first-step estimation has an error up to $\varepsilon = 10..30\%$, and after optimization $\varepsilon \leq 1\%$. 

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4. Practical example

As a practical example, let’s show the result of ‘optimal representation’ of evolution curve for Italian music (see Fig. 1) with 7 sinusoidal components. The list of components is given in Table 1. All the components have aliquant frequencies. The two first components in Table 1 are ‘overlapping’ each other: they form together a wide spectrum lobe at \( T_p = 100.98 \) on Fig. 3. The resulting approximation error (relative value) was about 9.3%.

<table>
<thead>
<tr>
<th>Number of component</th>
<th>Period, years</th>
<th>Relative amplitude</th>
<th>Phase (rad.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130.014</td>
<td>0.4748</td>
<td>-1.085</td>
</tr>
<tr>
<td>2</td>
<td>100.296</td>
<td>1.0000</td>
<td>1.680</td>
</tr>
<tr>
<td>3</td>
<td>69.028</td>
<td>0.6000</td>
<td>-1.961</td>
</tr>
<tr>
<td>4</td>
<td>60.868</td>
<td>0.4759</td>
<td>-0.8525</td>
</tr>
<tr>
<td>5</td>
<td>47.444</td>
<td>0.3019</td>
<td>2.8199</td>
</tr>
<tr>
<td>6</td>
<td>39.592</td>
<td>0.1798</td>
<td>1.0289</td>
</tr>
<tr>
<td>7</td>
<td>37.709</td>
<td>0.1425</td>
<td>1.5787</td>
</tr>
</tbody>
</table>

All the needed procedures of polynomial approximation, centering and smoothing, and also spectrum analysis were realized with the help of a special computer program ‘Waves_Ex’ (Waves Examination) derived by the author.

The results of curves analysis can be further used for building of mathematical dynamic models of appropriate socio-cultural processes.

References