Optimal design of high-efficiency retrodiffraction gratings for polarizing filters based on the rigorous coupled-wave analysis

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Abstract
This paper presents the fast and reliable genetic algorithm (GA) and simulated annealing algorithm (SA) optimization technique for high efficiency polarization-sensitive diffraction gratings utilizing the total internal reflection (TIR) in a lamellar configuration; diffraction characteristics were evaluated by the rigorous coupled-wave analysis (RCWA), and a rough estimation of grating parameters (period, duty ratio, depth) was made by using the Taguchi method, and then, the optimum grating parameters in order to achieve an ideal polarization-sensitive performance were finely obtained by GA and SA, respectively. As a result, the 1st-order diffraction efficiencies for TE and TM polarization were estimated up to 95.5%, 2.7% in a rough estimation and 99.9% and 0% in a fine estimation, respectively. It can be seen that both hybrid Taguchi-GA (HTGA) and SA (HTSA) showed a significant improvement with regard to its performance and computation time.

Keywords: Retrodiffraction, TIR, RCWA, Taguchi method, SA, GA

1. Introduction
Diffraction gratings play an important role in many optical applications, including optical telecommunications components, spectroscopy, display systems and the chirped pulse amplification (CPA) for a high power laser system [1-13]. Especially, diffraction gratings utilizing TIR within a metal-free groove are an integrated optical element expected to achieve a high performance at a specific retrodiffracted order [1-4]. Absorption of the incident light at the metallic surface diminishes the diffraction efficiency to 75 - 95%, depending on the material and wavelength of the incident light, which can result in the severe damages on the grating facets because this absorbed energy turns into heat [3, 8]. Therefore, the TIR-based dielectric gratings can be available to provide a metal-free solution and a high optical performance.

In general, diffraction gratings with a lamellar configuration consist of a periodically varying boundary of the period $P$ between two different media with refractive indices $n_0$ and $n_i$ as illustrated in Fig. 1, where the filling factor $F$, the depth $D$, the wavelength of incident light $\lambda$, the incident angle and $\theta_m$ the m-th order diffraction angle, respectively. The TIR-based grating equation may be expressed as [3, 4],

\[
\frac{n_i \sin(\theta_i) + n_0 \sin(\theta_m) + m \frac{\lambda}{P}}{P} = 0, \tag{1a}
\]

\[
n_0 \leq \frac{\lambda}{2P} < n_i, \tag{1b}
\]

Whereas the angular and spectral characteristics of gratings solely depend on Eq. (1), diffraction efficiency is dependent on the surface profile of gratings, which is calculated by RCWA [14-17]. In accordance with the TIR grating condition, it might be a big obstacle to obtain the optimum grating parameters, $P$, $F$ and $D$. At this time, a stochastic approach to design the grating parameters can be employed such as the Taguchi method, GA, SA and so
on [4, 7, 18-22]. The Taguchi method provides the optimal levels of significant parameters based on the signal-to-noise ratio (S/N ratio) and a statistical analysis of variance (ANOVA) of the design parameters, and GA and SA explore the function’s entire space and provide the global optimization of a given merit function used to evaluate the difference between the desired value and the calculated one [18-24]. However, GA and SA always take plenty of computation time in finding globally optimal conditions, which is not suitable for many engineering applications [20-22]. Thus, it is largely dependent on the starting values so that the optimization process becomes worse due to the bad interval of variables [23, 24]. To provide the reliable results and the economic computation for the parametric optimization many auxiliary techniques such as the hybrid optimization method connecting two different algorithms have been proposed [20, 22].

Many GA or SA optimization techniques have been applied to the several optical applications but mainly focused on the multi-layer systems [18, 19]. However, the optimum design of a high efficiency diffraction gratings based on TIR has not been well-introduced. To improve results achieved in the previous research, in this Letters, by use of HTGA and HTSA, the fast and reliable optimum design techniques for high efficiency polarization-sensitive TIR diffraction gratings are presented.

2. Optimization process

The HTGA and HTSA optimization presented in this paper is illustrated in Fig. 2; \(X = < P, F, D >\) can exists only in all set of admissible space \(\Omega\) and the starting point A, quasi-optimized point B and ideal point C. The whole optimization process is performed in this sequence; a rough estimation by use of the Taguchi method from A to B and a fine estimation by use of GA and SA from B to C. The Taguchi method narrows down all set of admissible space \(\Omega\) to the quasi-admissible space \(\Psi\), which provides the starting values for GA and SA and saves the computation time to find the optimum point within the narrow-downed search region. The whole space is defined as \(\Omega = \{ X(P, F, D) | 0.330 < P[\mu m] < 0.534, 0.1 < F < 0.9, 0.4 < D[\mu m] < 1.0 \}\) and as a result of a rough optimization by use of the Taguchi method, \(X(0.435, 0.5, 0.85)\) was obtained [4], and then, in terms of the S/N ratio and ANOVA of each parameter the narrow-downed quasi-admissible space \(\Psi = \{ X(P, F, D) | 0.4 < P[\mu m] < 0.5, 0.3 < F < 0.6, 0.8 < D[\mu m] < 1.0 \}\) was subdivided.

To solve the GA and SA optimization problem, the merit function (M.F) can be expressed as [20, 21],

\[
M.F = [f(X) - f(X^*)]^2, \quad \text{for all } X
\]  
\[f(X) = |DE_{-1,TE} - DE_{-1,TM}|, \quad \text{for } DE_{-1,TE} > DE_{-1,TM}
\]

where, \(f(X^*) = 1.0\) is a desired target value and \(f(X)\) is the current value. \(f(X)\) is a function of the difference between the desired target value and calculated diffraction efficiencies for TE (DE_{-1,TE}) and TM (DE_{-1,TM}) at the -1\(^{\text{st}}\) order. When the merit function comes down to zero,
the objective is achieved and optimal parameters are obtained [18, 19]. The RCWA conditions are tabulated in Table 1; the refractive index of the substrate \((n_i=1.65, \text{ heavy flinted glass})\), the wavelength \((\lambda=1.064 \ \mu m)\) and the incident angle of light \((\theta=45^{\circ})\) are assumed. The diffraction efficiency is defined as the fraction of incident monochromatic light diffracted into a specific order [11]. In this present work, for the numerical calculation, convergence was achieved in each case by including 101 (-50 to +50) space-harmonic components. Also, it is assumed that the sum of diffraction efficiencies at all harmonic orders keeps 100%, which indicates that the grating is lossless.

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Table 1. Optical and structural properties for analysis.

<table>
<thead>
<tr>
<th>(\lambda[\mu m])</th>
<th>(\theta_i)</th>
<th>(n_0)</th>
<th>(n_i)</th>
<th>(P[\mu m])</th>
<th>(F)</th>
<th>(D[\mu m])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.064</td>
<td>45.0(^{\circ})</td>
<td>1.0</td>
<td>1.65</td>
<td>(P)</td>
<td>(F)</td>
<td>(D)</td>
</tr>
</tbody>
</table>

3. Results

Based on the quasi-admissible space \(\Psi\) obtained by the Taguchi method, GA and SA optimization is performed in Matlab environment (PC, CPU 1.7GHz, RAM 1G byte). The starting values are set to \(X(0.435,0.5,0.85)\) found by a rough estimation and GA and SA start finding the optimal values \(X^*\) to make the merit function zero within the quasi-admissible space \(\Psi\), respectively. Some of the feasible values were obtained, \(X(0.457,0.400,0.902)\), \(X(0.464,0.391,0.890)\), and \(X(0.458,0.394,0.907)\) as a result of GA and \(X(0.456,0.402,0.908)\), \(X(0.461,0.405,0.888)\), and \(X(0.458,0.386,0.913)\) as a result of SA. Two optimizations took less than 20 min. for all cases even though RCWA was performed every time while finding the feasible values and no failure was found while running. It can be seen that all of the feasible values by two optimizations are within 2~3 % variation of each parameter and those diffraction characteristics as to the feasible values are almost identical, which can be understood that the global minimum is reached. The spectral response of the gratings designed by HTGA and HTSA is shown in Fig. 3. The -1\(^{st}\) order diffraction efficiencies for TE and TM polarization were estimated to be up to \(DE_{-1,TE}=95.3%\) and \(DE_{-1,TM}=2.7%\) for the Taguchi method [4], \(DE_{-1,TE}=99.9%\) and \(DE_{-1,TM}=0.0%\) for GA, and \(DE_{-1,TE}=99.9%\) and \(DE_{-1,TM}=0.0%\) for SA at \(\lambda=1.068 \ \mu m\), respectively. At a center of \(\lambda=1.064 \ \mu m\), the designed gratings provided the diffraction efficiency higher than 95% within 30 nm bandwidth. From Fig. 3, it was confirmed that GA and SA optimization are very helpful to provide the global minimum point of the merit function. Even though the Taguchi method can not make \(DE_{-1,TM}\) nearly zero and GA and SA can lower \(DE_{-1,TM}\) to zero by a fine parametric estimation. On the other hand, GA and SA optimization was applied without the Taguchi method within the whole space \(\Omega\). The starting values were set to the middle point of each parameter and \(X(0.458,0.745,0.689)\), \(X(0.458,0.739,0.677)\), \(X(0.458,0.784,0.745)\) for GA and
X(0.458, 0.831, 0.891), X(0.458, 0.592, 0.602), X(0.458, 0.742, 0.678) for SA were obtained, respectively. The gratings with those values were polarization-sensitive, but it took 3~4 times longer than that of HTGA and HTSA. Also, the obtained values were randomly distributed in the whole space except for parameter P, which can not be quite sure of where the global minimum is located. We considered that those designed gratings are not reliable because it may bring a radical change in its performance due to the small defects such as fabrication errors or material imperfection if the optimized values are a unique point. As a result, both HTGA and HTSA brought a great deal of improvement with regard to the polarization-sensitive performance, global optimization and computation time.

Fig. 3. Comparison of the spectral responses between the results of the Taguchi method only (Ref. [4]) and those of HTGA and HTSA: X(0.435, 0.5, 0.85) for Ref. [4] and X(0.457, 0.400, 0.902) for HTGA and X(0.436, 0.402, 0.908) for HTSA.

4. Conclusion
The fast and reliable design technique for the polarization-sensitive TIR gratings by using HTGA and HTSA has been presented. The TIR gratings designed by a fine estimation showed 99.9% of the polarization-sensitive performance and diffraction efficiency over 95% was distributed within 30 nm bandwidth at a center of \(\lambda = 1.064 \ \mu m\), which showed a significant improvement compared to that of the Taguchi method only. HTGA and HTSA were considered as the reliable optimization technique to obtain the globally optimal values and saved the computation time because the Taguchi method can provide the initial values and the narrow-downed search region. As a result, it is expected that HTGA and HTSA shows feasibility of the fast and reliable optimization approach to a technique for designing optical devices.

5. Acknowledgements
This work was supported by the Korea Science and Engineering Foundation (KOSEF) NRL Program grant funded by the Korea government (MEST) (No.R0A-2008-000-10065-0).

References