Absolute distance measurements based on the frequency comb of a femtosecond pulse laser

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Abstract

We describe absolute distance measurements performed by exploiting the frequency comb of a femtosecond laser as a precision wavelength ruler. The frequency comb is first stabilized to a Rb atomic clock of time standard. Then multi-wavelength interferometry integrated with a wavelength-scanning scheme is implemented by tuning an external cavity diode laser to the pre-selected frequency modes of the optical comb. The achieved wavelength uncertainty is 5.9 parts in $10^{12}$, which enables absolute distance measurements up to ~1 meter with overall uncertainty less than 20 nm.

Keywords: Absolute distance, interferometer, optical comb

1. Introduction

Laser interferometers based on homodyne or heterodyne principles are widely used for industrial distance measurements [1]. A single wavelength is used to measure a distance by continuously accumulating the incremental movement of the target mirror. This relative nature of distance measurement using a single wavelength yields a short non-ambiguity range that is just half the wavelength in use. To extend the non-ambiguity range to measure an absolute distance, the wavelength needs to be tuned discretely over to several different wavelengths to perform multi-wavelength interferometry, or be varied continuously between two boundaries to conduct wavelength-scanning interferometry [2]. These absolute distance measuring methods consequently require a tunable laser source that is capable of generating highly stable and accurate optical wavelengths over a wide tunable range.

The precision of absolute distance measurements is greatly affected by the individual accuracy of the used wavelengths. The wavelength of a laser source is usually calibrated with reference to well-defined absorption bands of atoms or molecules. This conventional wavelength calibration requires a complicated optical setup and is furthermore limited by available absorption bands. In recent years, the advent of femtosecond pulse lasers permitted facilitating the calibration of optical wavelengths with direct traceability to an atomic clock of a frequency standard. In line with the progress, the concept of optical frequency generator (hereafter abbreviated OFGenerator) was proposed for high precision absolute distance measurements. The OFGenerator is a monochromatic light source capable of providing accurate multiple wavelengths referenced to the frequency comb of a femtosecond pulse laser.

In this paper, we first describe the fundamental characteristics of the frequency comb of a femtosecond laser particularly with focus on how it can be stabilized with respect to an atomic clock of radio-frequency time standard. Then we introduce an OFGenerator developed to perform absolute distance measurement by exploiting the frequency comb as a precision wavelength ruler. Finally, we will demonstrate actual absolute distance measurements using the OFGenerator followed by detailed discussions on the precision achieved by this new attempt using the frequency comb.
2. The frequency comb and OFGenerator

The wavelength ruler needed for absolute distance measurement is constructed by stabilizing the frequency comb of a femtosecond laser. As illustrated in Fig. 1, the frequency comb is characterized by two independent parameters; the repetition rate $f_r$ and the carrier-envelope offset frequency $f_o$. In the frequency domain, $f_r$ represents the mode spacing between two adjacent frequency teeth. And $f_o$ denotes the frequency offset of the whole comb from the absolute zero frequency position, which is caused by the difference between the group and phase velocities of the pulses in the time domain. Since these two parameters $f_r$ and $f_o$ fall in the radio-frequency region of usually less than 1 GHz, an accurate wavelength ruler is established in the optical frequency region by stabilizing both the $f_r$ and $f_o$ to a well-established frequency standard working in the radio-frequency regime, such as a Rb atomic clock [3, 4].

![Figure 1. Frequency comb of a femtosecond laser.](image)

The stabilized comb is comprised of a large number ($10^5$-$10^6$) of quasi-monochromatic modes with a spacing of $f_o$, and each mode yields an extremely small optical power of 10 to 100 nW. To generate a single wanted wavelength with a sufficient power of several mW as required for the OFGenerator, a tuneable external diode laser is employed as the working laser. The output frequency of the diode laser is tuned consecutively to the selected modes within the comb. To precisely detect the actual output frequency of the diode laser during tuning, it is interfered with the comb which gives many beat notes of different frequencies. The lowest frequency beat $f_b$ is extracted through a low-pass filter and subsequently identified using a radio frequency counter. At the same time, the wavelength of the diode laser is also measured using a wavelength meter. Combining the two measured readings with known values of $f_r$ and $f_o$ leads the frequency of the working diode laser to be expressed as $f_w = nf_r + f_o + f_b$, in which $i$ is a large integer. The uncertainty of $f_w$ can be precisely determined with reference to the Rb clock used to stabilize the frequency comb. Finally, by tuning the frequency of the external working laser to produce the pre-assigned set of $i$ and $f_b$, any optical frequency required for absolute distance interferometry can be synthesized from the working laser.

3. Absolute distance measurement

The wavelength of light relates to the frequency as $\lambda = c/(nf)$ where $c$ is the speed of light in vacuum and $n$ is the refractive index of the medium within which light propagates. For a given wavelength $\lambda$, the distance to be measured can be expressed as $L = (\lambda/2)(m + e)$, where $m$ and $e$ denote an integer ($m = 0, 1, 2, \ldots$) and excess fraction ($1 > e \geq 0$), respectively. The excess fraction can be directly determined by analyzing the resulting temporal or spatial interference signals, while the integer part is not the case because of the 2$\pi$-ambiguity of the single-wavelength interferometer.

Multi-wavelength interferometry determines an absolute distance by employing multiple wavelengths [5, 6]. The absolute distance $L$ thereby can be written in the form of simultaneous equations as
where the subscript N indicates the total number of individual wavelengths in use. Since all of m_i must be positive integers, the absolute distance L can be determined numerically with excess fraction values, e_i (i = 0, 1, ..., N), that are obtained at each wavelength \( \lambda_i \). An attractive merit of this technique is that measurement uncertainty can reach the same level as that of relative displacement measurements. In order to determine a unique solution out of the preliminary solution sets in the numerical process, the distance should be estimated prior to measurement. Increasing the number of wavelengths in use with a well-defined manner or adapting the principle of wavelength scanning interferometry enables complete avoidance of the ambiguity problem encountered in single-wavelength interferometry.

4. Experiments

![Schematic diagram of the OFGenerator built in this investigation.](image)

Figure 2 shows a schematic diagram of the OFGenerator. The Ti:Sapphire femtosecond pulse laser emits a train of pulses of 35 fs duration with a central wavelength of 780 nm at a repetition rate of 81 MHz. To stabilize the frequency comb, the pulse repetition rate \( f_r \) was locked to an Rb clock by translating the output coupler of the oscillator. At the same time, the carrier-envelope offset frequency \( f_o \) was measured with a self-referencing \( f-2f \) interferometer and secured to the same Rb clock signal by adjusting the tilt angle of the cavity end mirror located after the prism pair. This scheme of frequency stabilization first proposed in 2000 [3, 4] enables direct observation and stabilization of \( f_o \) to a rf frequency standard without any third party stabilized laser source. An extra-cavity laser diode (ECLD) is adopted as a working laser and it provided an average power of 12 mW with a linewidth of less than 300 kHz. The output frequency of the ECLD is continuously tuned over the range of 765 to 781 nm. The working laser is then locked to the selected mode of the frequency comb. The beat \( f_b \) between the ECLD frequency and its closest mode is stabilized with respect to the Rb clock by feedback-control of the input current to the ECLD. A wavelength meter is used to make the working laser be tuned with a resolution of 8 MHz (0.016 pm in wavelength), which is rather coarse but accurate enough to obtain an unambiguous access to the selected optical comb mode.
Consecutive multiple wavelength generation is performed using the OFGenerator for absolute distance measurement. Figure 3 shows an exemplary test result on the frequency stability of the OFGenerator that is worked out in terms of Allan deviation. Frequency stability of the repetition rate $f_r$ of the comb yields $1.3 \times 10^{-12}$ and that of the beat note between the working laser and the optical comb is $5.8 \times 10^{-12}$ for 10 s of gate time, respectively. The resulting wavelength uncertainty of the OFGenerator is $5.9 \times 10^{-12}$, which is better than that of the I$_2$-stabilized He-Ne laser ($\sim 10^{-11}$) used as one of the national length standards.

A heterodyne interferometer was set up for absolute distance measurements as shown in Fig. 4. Two acousto-optic modulators (AOM) were used to generate slightly different optical frequencies for heterodyne phase detection. The light generated from the OFGenerator experiences two different optical paths (the reference and measurement) and recombined at a beam splitter. The optical path difference between two arms was around 1.2 m. Avalanche photo-detectors (APD) 1 and 2, were installed to detect the resulting interference signals. A precision phase-meter was used to measure the phase differences between two interference signals with four different wavelengths as required for absolute distance measurements based on multi-wavelength interferometry.

Finally, an actual example of absolute distance measurement using OFG is discussed in detail. Four wavelengths, 770.204349 nm, 779.953524 nm, 780.203668 nm, 780.206961 nm, were successfully generated from the OFGenerator. This particular set of four wavelengths was chosen in an optimized way to ease the initial estimation of the distance to be measured. The absolute distance of $\sim 1.2$ m was measured using the four wavelengths, and environmental parameters such as temperature, relative humidity, pressure, and CO$_2$ contents were monitored to compensate the refractive index of air during experiments [7]. The resulting measurement uncertainty of the absolute distance was found less than 20 nm in faithful accordance with the ISO-recommended guidelines [8].
5. Conclusion

The concept of optical frequency generator exploiting the frequency comb of a femtosecond laser was proposed and verified for the purpose of absolute distance metrology. The frequency comb was stabilized collectively to a Rb clock of frequency standard and an external cavity diode laser was consecutively locked to pre-selected modes of the comb. The multiple-wavelength interferometry using four different wavelengths was successfully demonstrated with an overall uncertainty of less than 20 nm for an absolute distance of 1.2 m. The absolute distance metrology proposed in this study would find its applications in various scientific and industrial fields due to its high precision and traceability to the well-defined international definition of time.

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References