The geometric dynamic errors of CMM in fast probing

Wei Jinwen\textsuperscript{1,2}, Chen Yanling\textsuperscript{2}, Guo Junjie\textsuperscript{1}

\textsuperscript{1} State Key Laboratory for Manufacturing Systems Engineering, Xi’an JiaoTong University
Xi’an, 710049, China
Tel: +086 0771 3273632 Fax: +086 0771 3273632 E-mail: my595@sina.com.cn

\textsuperscript{2} School of Mechanical Engineering, GuangXi University, Nanning, 530004, China
Tel: +086 0771 3232294 Fax: +086 0771 3232294 E-mail: weijw@gxu.edu.cn

Abstract
Static/quasi-static model of the geometric errors of Coordinate Measuring Machine (CMM) is effective in slow probing, but losses its compensation accuracy in fast probing due to the generation of dynamic errors. This paper presents a dynamic model for the geometric errors of CMM in fast probing with Least Squares method. Taking the 6 geometric errors of a CMM slide way and probing acceleration for the inputs, the positioning errors of probe tip can be decomposed into 7 components corresponding to the 7 inputs in this dynamic model. Experiments showed that the 2 angular error components of x slide way, \( \varepsilon_1(x) \) and \( \varepsilon_2(x) \) around y and z axis respectively, can induce significant dynamic errors that may alter up to 30% or higher of their corresponding static components at probe tip, especially in a CMM of low rigidity aerostatic slide way. So smoothening the sharp corners of the curves of geometric errors, especially of \( \varepsilon_y-x \) and \( \varepsilon_z-x \), is considered as a simple but effective and reliable method to improve the compensation accuracy of static/quasi-static model in fast probing.

Keywords: Coordinate measuring machine; geometric error; dynamic error; error compensation

1. Introduction
The geometric errors of CMM come from the imperfection of CMM slide ways, which are uneven, pitching and rolling rather than perfect geometries (Fig. 1). Moreover the deformations and deflections of low stiffness CMM parts can be another significant sources of the static errors of probe tip and are generally counted into equivalent geometric errors of its slide ways [1]. Based on quasi-rigidity assumption, geometric errors are calibrated under static or quasi-static state and mapped statically to the positioning error of probe tip with a static/quasi-static error model, not dealing with any dynamic process [2, 3]. However because CMM is far from static or quasi-static state in fast probing, those error models may degrade their compensation accuracy of positioning error [4]. More understanding on the geometric error model of CMM is necessary for both the sub-micron measurement and higher efficiency demands from industry. This paper studies the dynamic effects of geometric errors in fast probing and analyses their properties.

2. The dynamic model of geometric errors
Let \( e_{iX}(x) \) and \( g_{iX}(x) \) symbol the \( i^{th} \) positioning error component and the \( i^{th} \) static error component in x direction (denoted by the subscript), both originating from the \( i^{th} \) geometric error component of x slide way, of probe tip respectively. As an example to illustrate the dynamic effects in the geometric errors of x slide way, a model of bridge CMM (Down looking in Fig. 1, \( a+b=x_2-x_1 \)) is shown in Fig. 2, in which the bridge is regarded as a mass point \( m \) attached to stiff beam \( L \). The air cushions, which linked the stiff beam to x slide way, are taken for 2 lumped springs of rigidity \( k \). The vibration equation (disregarding damping) is:

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\[
\begin{align*}
&\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{e}_X \\ \ddot{e}_Y \end{bmatrix} + k \begin{bmatrix} -\frac{a^2+b^2}{L^2} & 0 \\ 0 & \frac{a^2-b^2}{L} \end{bmatrix} \begin{bmatrix} e_X \\ e_Y \end{bmatrix} = k \begin{bmatrix} -\frac{a^2-b^2}{L} & 0 \\ 0 & \frac{a^2+b^2}{L^2} \end{bmatrix} \begin{bmatrix} \xi_Y(t) \\ \xi_Y(t) \end{bmatrix} \end{align*}
\]

where the geometric errors of \( x \) slide way (Fig. 2) are:
\[
\begin{align*}
\{\delta_Y(t)\} &= [a \times \delta_Y (2) + b \times \delta_Y (1)]/(a + b) \\
\{\xi_Y(t)\} &= [\delta_Y (2) - \delta_Y (1)]/(a + b)
\end{align*}
\]  

Because (1) is uncoupled, \( e_X \) is simulated alone with the input of \( \xi_Y(t) \) in Fig. 4f. Depending on parameters \( m/k \) and \((a^2+b^2)/L^2\), the potential dynamic effects in the positioning error of probe tip could be actually activated by the quick fluctuation of the geometric errors in fast probing (Fig. 3c), and named as GDE (Geometric Dynamic Error) accordingly. It is rational to presume that:

1) The GDE in \( e_Y \) is too insignificant to be considered (as those in Fig. 3a ∼ b) while \( e_X \) contains relatively remarkable GDE (Fig. 3c) when \( a^2+b^2 \ll 2L^2 \). Likewise the GDE of the geometric errors of \( y \) and \( z \) slide way can be negligible due to the relative small mass of the Carriage and Spindle in CMM bridge;

2) If the bridge moves at a constant speed \( v \) on \( x \) slide way that had a geometric error \( \delta_Y(x)=A_\delta \sin(\omega \cdot x) \), then \( \delta_Y(x)=A_\delta \sin(\omega v t) \)

Therefore geometric errors are modulated by the probing speed of CMM.

3) The accuracy of geometric error compensation with static/quasi-static model might degrade in fast probing.

3. The GDE detection by experiments

When CMM probes quickly on \( x \) slide way, the error components of the positioning error of probe tip mainly include: 1) static errors that originated from the 6 geometric errors \{\( \delta(x), \dot{\delta}(x) \}\}, where \{\( \delta(x)\)=\{\( \delta_Y(x), \delta_Y(x), \delta_Z(x) \)\} and \{\( \dot{\delta}(x)\)=\{\( \xi_Y(x), \xi_Y(x), \xi_Z(x) \)\} symbolize the translation errors and angular errors of \( x \) slide way respectively; 2) acceleration error and stochastic component; 3) GDE. Perhaps GDE had once been taken for a kind of stochastic error before, but they could be separated from stochastic errors and compensated effectively if enough knowledge of them had been obtained. Those error components in the positioning error of probe tip are traced to their error sources by RLS method, in which CMM bridge is regarded as a system of 7 inputs: \{\( u_1, ..., u_7 \)=\{\( \delta_X(t), \delta_Y(t), \delta_Z(t), \xi_X(t), \xi_Y(t), \xi_Z(t), a(t) \)\} and 3 outputs: \( \{e\} = \{e_X(t), e_Y(t), e_Z(t)\} \). This 7-3 dynamic system can be taken apart into three 7-1 subsystems in terms of the 3 irrelative outputs \( \{e_X(t), e_Y(t), e_Z(t)\} \), and modeled by RLS (Recursive Least Square) method respectively, i.e.

\[
e_X(n) = \sum_{j=0}^{N} \sum_{i=1}^{7} h_{i,j} u_i(n-j) - \sum_{j=1}^{N} h_{i,j} e_X(n-j) - \xi_X(n)
\]  

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where: $H_X=\{h_{1, N},..., h_{7, N}, ...., h_{1, 0}, ..., h_{7, 0}, h_{0, N}, ..., h_{0, 1}\}^T$ are the coefficient vector of the system. For an arbitrary element $h_{i,j}$, the first subscript $i=1~7$ denoted the $i^{th}$ input; $i=0$ denoted output; the second subscript $j=0~N$ denoted the lag number. $\xi(n)$ is a white noise serial. The RLS estimation of $H_X$ is

$$\hat{H}_X(n+1)=\hat{H}_X(n)+\frac{P_X(n)u_X(n+1)[|e_X(n+1)|-q^T_X(n+1)\hat{H}_X(n)]}{1+q^T_X(n+1)P_X(n)u_X(n+1)}$$

where: $P_X(n+1)=P_X(n)-\frac{P_X(n)u_X(n+1)q^T_X(n+1)P_X(n)}{1+q^T_X(n+1)P_X(n)u_X(n+1)}$.

$$q^T_X(n+1)=[u_1(n), ... , u_1(n-N+1),... , u_7(n), ... , u_7(n-N+1), -e_X(n), ... , -e_X(n-N+1)]$$

$\hat{H}_X(0)=[0]_{8N\times1}$; $P_X(0)=c^2I_{8N\times8N}$, $c$ is a large constant e.g. $c=10^8$. The positioning error component of the $i^{th}$ input is evaluated by

$$\epsilon_i^X(n)=e_X|_{um=0,m \neq i} = \sum_{j=0}^{N} h_{i,j}u_j(n-j) - \sum_{j=1}^{N} h_{0,j}-\epsilon_J^X(n-j)$$

and $GDE_i^X(n)=\epsilon_i^X(n)-g_i^X(n)$, $i=1, ..., 6$ (5)

The task in the experiment is to check whether or not $\epsilon_i^X(t)=g_i^X(t)$. The CMM used in the experiment was a bridge-type DMC655 made by SiYuan Precision Engineering Co.Ltd, China (Fig. 1). The coordinate and actual displacement of probe tip were read by the Renishaw® scale of the CMM and a laser interferometer, MI1500 made by SIOS® Co.Ltd, Germany, respectively. The probing acceleration of CMM bridge was measured by an accelerometer (ADXL103 inside). The control computer had ensured the synchronous acquisition of the data from the 3 equipments by integrating both the drivers of the MI1500 and the accelerometer into CMM software. The processes of the experiment are: 1) Statically calibrate the geometric errors of $x$ slide way; 2) Synchronously collect the 3 data serials: the coordinate $x(t)$; the displacement $x_{\text{laser}}(t)$ of probe tip; and the probing acceleration $a(t)$ of CMM bridge, in a fast probing test. Then $\{\tilde{x}(x), \tilde{x}(x)\}_6$ are interpolated by $x(t)$ to obtain $\{\tilde{x}(t), \tilde{x}(t)\}_6$; while the positioning error of probe tip is $e_X(t)=x_{\text{laser}}(t)-x(t)$; 3) Decompose $e_X(t)$ with (5) to obtain each error component $\epsilon_i^X(t)$.

4 Experimental results

Figure 4 shows the results of the experiment. A group of identified parameters $H_X$ from those experimental data are showed in Table.1. The experiment illustrated: 1) Acceleration is an important contributor to $e_X(t)$. The acceleration error component, dominating over 60% of $e_X$ at the stage of accelerating or decelerating the CMM, testified one of the necessary sources of a remarkable GDE: low rigidity of $x$ aerostatic slide way; 2) $\epsilon_Y(x)$ and $\epsilon_Z(x)$ are the most “dynamic” among the 6 geometric errors and can cause remarkable GDE in $e_X(t)$ under certain conditions. The other 4 geometric components exhibited “dynamic effects” through the corresponding high-ordered coefficients $h_{i,j}$ ($i<5$) either. Yet those “dynamic effects” are far smaller than those of $\epsilon_Y(x)$ and $\epsilon_Z(x)$ and can be negligible hereby. Two pairs of error components, $\{\epsilon_Y^1(t), g_Y^1(t)\}$ of $\epsilon_Y(t)$ and $\{\epsilon_Y^2(t), g_Y^2(t)\}$ of $\epsilon_Z(t)$, are shown in Fig.5, in which the ratio $\eta^j(t)=GDE^j(t)/g_X^j(t)$, $g_X^j \neq 0$ is an indices describing the difference between $e^j_X(x)$ and $g_X^j(x)$ respectively: $max(\eta^y)=33.03\%$; $max(\eta^z)=12.66\%$.

The experiment had also indicated that the intensity of $GDE_Y^y$ and $GDE_Z^z$ mainly depended on the following factors: 1) Probing speed and the smoothness of curves $\epsilon_Y \sim x$ and $\epsilon_Z \sim x$. The undulation of the geometric errors, modulated by probing speed in (3), is the exciter of GDE. The maximum $\eta^y_i$ generally occurred at the sharp turning of curves $\epsilon_Y \sim x$ or $\epsilon_Z \sim x$ in fast probing, and increased with the probing speed; But when $v \leq 30mm/s$, $GDE \rightarrow 0$ had been found for the DMC655 in the experiment. Thus $30mm/s$ is the critical speed, below which the $GDE$s of the DMC655 are negligible; 2) y coordinates i.e. the position of the carriage and the spindle, which take about 1/4 of the mass of the whole CMM bridge, on CMM beam. As indicated in the
experiments, the coefficient $h_{6,0}$ of $\varepsilon_{d}(x)$ increased in direct proportion with $y$ coordinates approximately, and the high-ordered coefficients $h_{6,j}, j>0$, tends to increase with $y$ coordinates either in the range $y \in [120mm, 500mm]$. Comparatively $Z$ coordinates has less influence on $GDE^{z,y}$ due to the relatively small mass of CMM spindle; 3) The rigidity of aerostatic slide way and CMM bridge. The static deformation and deflection of CMM parts especially of air cushions are generally incorporated into geometric error components in geometric error calibration. So the dynamic deformation and deflection of air cushions in fast probing could induce the corresponding GDE in $\varepsilon_{x}$. Aerostatic cushions are the elastic components of the system in Fig. 2, so a lower rigidity of aerostatic slide way can cause significant GDE (Table 2).

![Fig. 4. The acquired data in experiment (sampling rate $f_{s}=44.5$Hz).](image)

![Fig. 5. The GDE at $v=55.7$mm/s, $y=120$mm, $z=250$mm.](image)

![Fig. 6. (a) Smoothening the curve of a geometric error component; (b) The smoothening of $\varepsilon_{t}(t)$.](image)

Table 1. The identified parameters $H_{s}$.

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<tr>
<th>$j$</th>
<th>$h_{0,j}$</th>
<th>$h_{1,j}$</th>
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<th>$h_{3,j}$</th>
<th>$h_{4,j}$</th>
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</tr>
</tbody>
</table>

units: $h_{6,0}$, $r^{-1}$; $h_{6,j}$, $\mu m/10^3 rad$; $h_{7,j}$, $\mu m/10^3 mm$. Number of effective aerostatic cushions: 3*3=9.

Table 2. Another $[h_{5,j} h_{6,j}]$

<table>
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<th>$h_{5,j}$</th>
<th>$h_{6,j}$</th>
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</table>

Number of effective aerostatic cushions: 3*2=6.
5. Discussions and conclusion

Static/quasi-static error model plays an important role in the compensation of geometric errors in static or quasi-static measurement. But the 2 components, $\varepsilon_i(x)$ and $\delta_i(x)$, can bring dynamic errors and degrade the accuracy of error compensation in fast probing. Depending on those factors mentioned above, the maximums of the 2 GDEs could reach up to 30% of their corresponding static components in the positioning errors of probe tip.

Though GDE seems insignificant when compared with other dynamic errors e.g. acceleration error, the compensation of GDE is still necessary at least for the purpose of pure academic discussion. Theoretically the RLS model of (5) could be employed to estimate GDE, however its uncertainty from noises might impair its predominance over static/quasi-static error model. For example, it was found in the experiment that the RLS model sometimes failed to convergence when $v>100\text{mm/s}$ or $\text{acc}>1000\text{mm/s}^2$, perhaps due to the rise of color noise in error measurement. It is even unconvinced that $e'_X(t)$ is always more veracious than $g'_X(t)$, or vice versa, in Fig. 5. Thus the error compensation with RLS model is not recommendable. Yet making a compromise between these 2 models might be a good scheme for the compensation of geometric errors.

CMM bridge tends to move along a straight-line trajectory under inertial force in fast probing, without vibration if damping is high enough. For a simple, reliable and more veracious error compensation, it is advisable to smoothen the curves of $\delta(x)$~$t$, as well as the curves of $\varepsilon(x)$~$t$ probably, according to probing speed and $y$ coordinates (Fig. 6). Three residual errors of Fig. 4d, after the compensation by RLS model $e_X(t)-\sum_{i=1}^{7}e'_X(t)$, static model $e_X(t)-e'_X(t)-\sum_{i=1,5,6}g'_X(t)$ and its smoothened version $e_X(t)-e'_X(t)-\sum_{i=1,5,6}g'_X(t)$ respectively, are illustrated in Fig. 7. It is found that there is a little improvement of error compensation by the “smoothened” version of static model over its prototype and RLS model. Therefore depending on a good smoothening of $\delta$~$t$ and $\varepsilon$~$t$ curves, it is possible to achieve accurate compensation of geometric error by the “smoothened” static/quasi-static error model in fast probing. Moreover according to (1), enhancing the rigidity of the air cushions of $x$ slide way will help to reduce both acceleration error and GDE.

6. Acknowledgements

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