Laser Doppler visualization of velocity fields with eliminating the influence of multi–particle scattering

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Abstract

The method of laser Doppler visualization and measurement of a velocity field in flows with suppression of influence of a multi-particle scattering is discussed. The investigated section in the flow is illuminated with a laser sheet. The new concept is based on reception of the paired differences of frequency-demodulated normalized images of the laser sheet in light beams scattered in various directions.

Keywords: Laser Doppler velocimetry, laser visualization of velocity fields, Doppler Global Velocimetry (DGV), multi–particle scattering

1 Introduction

One of the major applications of lasers is optical diagnostics of flows of gas and condensed fluids [1-2]. Laser Doppler visualization and measurement of a field of velocities has been offered in [3]. This method is based on illumination of section of the flow by the laser sheet which image in the frequency-demodulated scattering light displays distribution of Doppler shift of frequency.

Doppler shift is proportional to the projection of the velocity vector to the direction set by the difference of the wave vectors $\mathbf{k}_s - \mathbf{k}$, where \mathbf{k} – the wave vector of the laser sheet, \mathbf{k}_s – the wave vector of the scattered light beam. Therefore distribution of Doppler shift represents field of the corresponding projection of velocity vector in the image of section of a flow. A gas absorption cell is proposed for use as an optical frequency demodulator for visualizing and measuring velocity fields of flows in real-time. The slope of the absorption characteristic of the gas cell is used as a discrimination curve for frequency demodulation of a scattered-light image of a section of a flow irradiated by a laser sheet. The possibility of using various substances having different optical properties as optical frequency demodulators is underlined. For parameter control of the frequency transfer function in gas cells it is proposed to use the methods of nonlinear optics and spectroscopy of the saturated absorption.

The optical Doppler processor (ODP) with coherent feedback is used to convert frequency shifts into intensity variations in [4-5]. In one implementation, the optical semiconfocal resonator as a converter frequency/intensity and the slope of its resonant transfer function as a frequency discrimination curve has been used. The optical Doppler processor is matched to the laser.

In [6] it was suggested to use molecular absorbing cell on iodine as the converter

frequency/intensity. Normalization of the image carried out by the second CCD camera is registering the reference image of a light plane directly. Normalization pixel–wise division of the signal image allows excluding the influence of fluctuations of an intensity of the scattered light on result of measurement.

Method for laser Doppler measurement of velocity field using acousto–optical frequency switching of the light plane is described [7]. This method is based on fast switching of the laser light illuminating the light section by a well defined frequency shift within one slope of the absorption line of the gas cell. Normalization is achieved by the ratio of frequency-shifted images without any reference camera.

In [8] sine wave frequency modulation of a light sheet is described. It allows increasing the temporal resolution of measurements. A velocity field is determined through the ratio of harmonics of the first and second order arising at transformation of the frequency modulated scattered light by transfer function of the converter frequency/intensity. The converter is carried out as an absorbing molecular cell.

Common disadvantage of all known methods of laser Doppler visualization and measurement of the velocity field which have historically received name Doppler Global Velocimetry (DGV) is influence of a many-particle scattering in a light sheet on result of measurements. Influence of a many-particle scattering is especially strongly shown at the big concentration of scattering particles in a flow. The possibility how to eliminate the dependence of DGV results from multi - particle scattering will be discussed in this paper.

2. Conception of elimination of multi–scattering influence in DGV–system

Fig. 1. Configuration of light beams in the wave vector space for $\mathbf{k}_{s1} = -\mathbf{k}_{s2}$.

The configuration of light beams in the wave vector space in Fig. 1 shows a method of eliminating a multi-particle scattering influence on a result of a velocity field measurement. Let the laser sheet is formed by two spatially matched laser beams sequentially succeeding in time, with frequency difference by a known value 2Ω . Frequency of the beams switching $\tilde{\omega}$ should not be less than Nyquist frequency depending on width of a spectrum of investigated process. The laser sheet is formed by an incident light field having a wave vector **k**. Wave vector of a laser sheet **k** . The light fields scattered by a particle *n* in directions, orthogonal a laser sheet are registered. Wave vectors of the scattered light fields \mathbf{k}_{s1} and \mathbf{k}_{s2} are orthogonal to the wave vector **k** of the laser sheet, thus $\mathbf{k}_{s2} = -\mathbf{k}_{s1} = \mathbf{k}_s$.

Frequency demodulation of the scattered beams are fulfilled by converters

Fig. 2. Symmetrical transmitting function of a frequency-to-intensity converter.

frequency/intensity 1-2. Images of the laser sheet in frequencydemodulated light are registered by CCD cameras 3-4. After that the images are processed on a computer.

The converter frequency/intensity has symmetric transfer function which examples are shown on Fig. 2. Let's choose working points P_1 and P_2 symmetrically one with regard to a central frequency in linear parts of the transmitting function as discrimination curves. Two images of the particle n in scattered light are subsequently registered by the camera synchronized with the sequence of the incident laser beams. In turn, either the right or the left slope (vicinity of working points P_1 or P_2) of the linear sections of the transmitting function of the frequency-tointensity converter are used. Let's write the expressions for the complex amplitudes of the beams scattered by particle n , the frequencies of which correspond to either the left or the right slope:

 $\hat{E}_n(\mathbf{k}_{s1}) = [1 + \text{sgn}(\sin \tilde{\omega}t)] \hat{E}_n(\Omega, \mathbf{k}_{s1}) + [1 - \text{sgn}(\sin \tilde{\omega}t)] \hat{E}_n(-\Omega, \mathbf{k}_{s1})$ $Slope.$
 $\hat{=}$ ($\hat{=}$) $\hat{=}$ ($\hat{=}$) $\hat{=}$ ($\hat{=}$) $\hat{=}$ ($\hat{=}$) $\hat{=}$, where $\widehat{=}$

$$
\widehat{E}_{n1}(\Omega, k_{s1}) = AS_{n1} \exp \{j[\omega_0 + \Omega + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k})]t + \varphi_n \} +
$$
\n
$$
+ A \sum_{m} S_{nm1} \exp \{j[\omega_0 + \Omega + \mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k}) + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})]t + \varphi_{nm} \},
$$
\n
$$
\widehat{E}_{n1}(-\Omega, k_{s1}) = AS_{n1} \exp \{j[\omega_0 - \Omega + \mathbf{v}_n]t + \varphi_n \} +
$$
\n(1)

$$
\widehat{E}_{n1}(-\Omega, k_{s1}) = AS_{n1} \exp \{j[\omega_0 - \Omega + \mathbf{v}_n]\} + \n+ A \sum_{m} S_{nm1} \exp \{j[\omega_0 - \Omega + \mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k}) + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})]\} + \varphi_{nm}\}.
$$
\n(2)

Eq. (1) and Eq. (2) describe the light fields the light field scattered by particle n in the direction of wave vector \mathbf{k}_{s1} , taking into account the contribution of scattered light that is incident on particle *n* from the next particle *m*. Here *А* is amplitude of incident light field with wave vector **k**; S_{n1} is a scattering function of particle *n* in a direction of a wave vector **k**_{s1}; S_{nm1} is scattering function of particle *n* in a direction of the wave vector **k**_{*s*1} for light incident on particle *n* from particle *m*; \mathbf{v}_n is a velocity of particle *n*; ω_0 is a laser light frequency; Ω is a frequency shift, corresponding to the working point on the slope of the transmitting function of the frequency-to-intensity converter that serves as discrimination curve; φ_n is a light wave phase, determined by the position of particle *n* in the light sheet; \mathbf{v}_m is a velocity of next particle *m*; \mathbf{k}_{nm} is a wave vector of light wave, scattered by particle *m* in the direction of particle *n*; $\mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k})$ is a Doppler frequency shift in light wave scattered by particle *m* in the direction of particle *n*; $\mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})$ is a Doppler frequency shift of the light wave with wave vector \mathbf{k}_{s1} , scattered by particle *n* from the incident light field that has already been scattered by particle m ; φ_{nm} is a light wave phase, determined by the position of particle *m* in the light sheet. After optical frequency–demodulation and frequency–to–intensity conversion the camera registers sequentially in time the following images of the *n* particle:

 $i_n(\mathbf{k}_{s1}) = [1 + \text{sgn}(\sin \tilde{\omega} t)]i_{n1}(\Omega, \mathbf{k}_{s1}) + [1 - \text{sgn}(\sin \tilde{\omega} t)]i_{n1}(-\Omega, \mathbf{k}_{s1}),$

where

where
\n
$$
i_{n1}(\Omega, \mathbf{k}_{s1}) = \xi A^2 S_{n1}^2 [\Omega + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k})] + \xi A^2 \sum_m S_{nm1}^2 [\Omega + \mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k}) + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})], \quad (3)
$$

$$
i_{n1}(\Omega, \mathbf{k}_{s1}) = \xi A^2 S_{n1}^2 [\Omega + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k})] + \xi A^2 \sum_m S_{nm1}^2 [\Omega + \mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k}) + \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})], \quad (3)
$$

$$
i_{n1}(-\Omega, \mathbf{k}_{s1}) = \xi A^2 S_{n1}^2 [\Omega - \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k})] + \xi A^2 \sum_m S_{nm1}^2 [\Omega - \mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k}) - \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})]. \quad (4)
$$

Here ξ is a conversion factor of the frequency/intensity converter. It is assumed that the intensity at the output of the converter is proportional to the frequency shift with regard to the working point. The term in the sum takes into account the contribution of multi-particle single scattering from the next particles *m* and determines the error in DGV field measurements at the point determined by the position of particle n . This partial contribution to the measurement error contains the Doppler frequency shift in light, scattered by particle *m* in the direction of particle *n*, proportional to the velocity projection \mathbf{v}_m of particle *m* on the difference wave vector $\mathbf{k}_{nm} - \mathbf{k}$, $\mathbf{v}_m(\mathbf{k}_{nm} - \mathbf{k})$, and the Doppler frequency shift, proportional to the velocity projection \mathbf{v}_n of particle *n* on the difference wave vectors $\mathbf{k}_{s1} - \mathbf{k}_{nm}$, $\mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{nm})$. These images are stored and are subjected to simple linear transformations.

As a result of subtraction of the second image (13) from the first one (12), one can obtain
\n
$$
i_{n12} = i_{n1}(\Omega, \mathbf{k}_{s1}) - i_{n1}(-\Omega, \mathbf{k}_{s1}) = 2\xi A^2 \left\{ \left(S_{n1}^2 + \sum_m S_{nm1}^2 \right) \mathbf{v}_n (\mathbf{k}_{s1} - \mathbf{k}) + \sum_m S_{nm1}^2 \mathbf{v}_{mn} (\mathbf{k}_{nm} - \mathbf{k}) \right\}.
$$
\n(5)

Addition of images (3) and (4) gives

$$
i_{n1}(\Omega, \mathbf{k}_{s1}) + i_{n2}(-\Omega, \mathbf{k}_{s1}) = 2\xi A^2 \left(S_{n1}^2 + \sum_m S_{nm1}^2\right)\Omega.
$$
 (6)

After normalizing the signal image (5) using the reference according to Eq. (6), will be ollowing:
 $\sum_{n} S_{nml}^2 \mathbf{v}_{mn} (\mathbf{k}_{nm} - \mathbf{k})$ the following:

$$
\widetilde{i}_{n1}(\mathbf{k}_{s1}) = \frac{i_{n1}(\Omega, \mathbf{k}_{s1}) - i_{n1}(-\Omega, \mathbf{k}_{s1})}{i_{n1}(\Omega, \mathbf{k}_{s1}) + i_{n1}(-\Omega, \mathbf{k}_{s1})} = \frac{1}{\Omega} \mathbf{v}_{n}(\mathbf{k}_{s1} - \mathbf{k}) + \frac{\sum_{m} S_{nm1}^{2} \mathbf{v}_{mn}(\mathbf{k}_{nm} - \mathbf{k})}{\Omega \left(S_{n1}^{2} + \sum_{m} S_{nm1}^{2} \right)}.
$$
\n(7)

Thus, using a frequency-to-intensity converter having a symmetrical transmitting function and single registering camera only, one can obtain the normalized frequencydemodulated image of the particle *n* that is formed in scattered light with wave vector **k***s*¹ according to Eq. (7). Using an analogous technique we obtain the normalized frequency-

demodulated image of the laser sheet in scattered light with wave vector
$$
\mathbf{k}_{s2}
$$
:
\n
$$
\tilde{i}_{n2}(\mathbf{k}_{s2}) = \frac{i_{n2}(\Omega, \mathbf{k}_{s2}) - i_{n2}(-\Omega, \mathbf{k}_{s2})}{i_{n2}(\Omega, \mathbf{k}_{s1}) + i_{n2}(-\Omega, \mathbf{k}_{s1})} = \frac{1}{\Omega} \mathbf{v}_n(\mathbf{k}_{s2} - \mathbf{k}) + \frac{m}{\Omega} \left(S_{n2}^2 + \sum_{m} S_{nm2}^2 \right)
$$
\n(8)

Here S_{n2} is a scattering function of the particle *n* in a direction of a wave vector \mathbf{k}_{s2} ; S_{nm2} is a scattering function of particle *n* in a direction of the wave vector **k***s*² for light incident on particle *n* from particle *m*. Then the difference of the normalized frequency-demodulated image images (7) and (8) is calculated. In case of identical scattering coefficients 2 $-c^2$ 2 $S_{n1}^2 = S_{n2}^2 = S_n^2$ and $S_{nm1}^2 = S_{nm2}^2 = S_n^2$ 2 $S_{nm1}^2 = S_{nm2}^2 = S_{nm}^2$ in the directions of **k**_{s1} and **k**_{s2} this difference is:

$$
\widetilde{i}_{n1}(\mathbf{k}_{s1}) - i_{n2}(\mathbf{k}_{s2}) = \frac{1}{\Omega}(\mathbf{k}_{s1} - \mathbf{k}_{s2}).
$$
\n(9)

For particles of the random form the influence of a multi-particle scattering should decrease due to statistical averaging on ensemble during an exposition. Under the multiplication of Eq. (9) by known value Ω , one can obtain the following:

 $\tilde{i}_{n1}(\mathbf{k}_{s1}) - \tilde{i}_{n2}(\mathbf{k}_{s2})\Omega = \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{s2})$ $\tilde{i}_{n1}(\mathbf{k}_{s1}) - \tilde{i}_{n2}(\mathbf{k}_{s2})\Omega = \mathbf{v}_n(\mathbf{k}_{s1} - \mathbf{k}_{s2}).$ (10)

As evident from Eq. (10), the influence of a many-particle scattering on result of measurement of a field of velocity can be excluded. Thus the linear scale of measurements and expansion of a dynamic range is provided. The frequency shift 2Ω of the switched laser beams alternately forming the light sheet is chosen equal to half the frequency interval between the working points in linear sections of the symmetrical transmitting function. The required subtraction and summation of frequency–demodulated images is realized by a computer.

3. Conclusion

 $_1$ (**k**₃₁) – i_{n2} (**k**₅₂) = $\frac{1}{\Omega}$ (**k**₅₁ = $\frac{1}{\Omega}$ (**k**₅₁ = $\frac{1}{\Omega}$ (**c**) statistical averaging on et solution of Eq. (9) by known value Ω , one identication of Eq. (9) by known value Ω , one influe The new concept of full or partial exception of influence of a many-particle scattering at DGV measurements of a velocities field in flows is presented. Our approach is based on reception of a pairwise difference of normalized frequency-demodulated images of a laser sheet in light beams scattered in various directions. In case when directions of the scattered beams pair are symmetric as regard to a light plane and scattering particles have spherical symmetry, the exception of influence of a multi-particle scattering is full. For particles of the random form the influence of a multi-particle scattering should decrease due to statistical averaging on ensemble during an exposition.

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