Fractal geometry surface modeling and measurement for musical cymbal surface texture design and rapid manufacturing

Jack Zhou\(^a\), Ananth Vas\(^a\) and Denis Blackmore\(^b\)

\(^a\) Department of Mechanical Engr. and Mechanics, Drexel University
Philadelphia, PA19104, USA

\(^b\) Department of Mathematical Sciences, New Jersey Institute of Technology
Newark, NJ 07102, USA

Tel.: + 1-215-895-1480 Fax: +1-215-895-1478 E-mail: zhoug@drexel.edu

Abstract

This paper aims at an equation based simulation, CAD modeling and manufacturing of a fractal surface embedded on a musical cymbal. This study is a proof-of-concept of a new method of complex-surface characterization, design and manufacturing using an equation-based approach. A cymbal shape was chosen to carry the fractal profile because generally most musical cymbals have an inherent broken surface pattern that is created to enhance the resulting musical quality. A fractal surface cymbal model was developed; a point cloud representation of the cymbal was generated from Matlab; a surface model was built and processed in image data processing software Imageware; a solid model was completed in CAD software PRO/E; a rapid prototype of the fractal surface cymbal was fabricated; and MasterCam software was used to generate CNC codes and simulate the CNC machining process. The effect of the variation of the parameters of the equation based surface is also shown by varying the topothesy and fractal parameter of the surface.

Keywords: Fractal geometry, surface modeling, rapid prototyping, cymbal surface

1. Introduction

One of the most common phenomena in nature is the fractal pattern. Fractal defines the quality of a shape that is constructed recursively, meaning that at all scales of magnification the shape appears similar. Often fractals are used to describe objects, specifically surfaces, which are “infinitely” complex. Benoit Mandelbrot, best known as the "father of fractal geometry" first coined the term “fractal” in 1975 [1]. Found in nature, fractals are the basis of complex systems, such as snow and clouds. Surfaces that cannot be described or recreated using classical Euclidean geometry are often described using fractal geometry [2]. Fractal surfaces are often naturally occurring self-affine surfaces that have in recent times, been used in various research areas such as chaos theory, electromagnetic scattering, musical synthesis, natural surface characterization, economic analysis, [3]. The type of sound a cymbal produces is directly related to its manufacturing process and resulting surface texture. Manufacturers must be able to produce a highly intricate surface structure to enhance the overall sound created when the cymbal is struck. To achieve the appropriate tone from a musical cymbal, the surface must have small indentations in a non-uniform array. Assuming that it is possible to embed a specific fractal pattern within the surface, we can alter the equation to create very specific musical outputs. Researchers have endeavored to use equations to describe and construct fractal surfaces for the last several decades [4, 5, 6, 7]. This approach to creating cymbals would allow a CAD/CAM system to be used for the actual manufacturing of the cymbals. Today, almost all cymbals are formed in the same manner, where the most critical part of the process is dependant on the skill of human workers. Because of this, no two cymbals manufactured in this manner will ever sound exactly alike. The significance of this research essentially lies in its uniqueness with respect not only to the
goal of creating an artificial fractal surface, but also in the novelty of the equation based-methodology used to create such a complex surface. Based on the literature survey both these goals are fairly unique in that barring one publication by Ruello and Blanco-Sánchez [8]; no one has attempted to create an artificial fractal surface. The advantages of this equation based methodology combined with CAD/CAM technology are: 1) A surface based on a mathematical equation has very little error and noise value as compared to hammer generated or scanned surfaces. 2) Fractal surfaces in particular are not usually amenable to scanning and other reverse engineering methods; our method models them quite effectively. 3) Altering the variables in the fractal equation will alter the acoustical characteristics, and then we can market different types of acoustics that can be reliably created to suit specific musical needs. 4) Our methodology can be reliably used to model any anisotropic and complex surface as long as it can be represented by a specific equation. 5) Another principal goal of the research is to establish optimal parameters for machining this surface using CNC milling or other methods such as freeform deformation [11] or Rapid prototyping etc [10].

2. Research methods
2.1 Mathematical Models. The mathematical model in this research is essentially a fractal equation which is a modification of a previously proposed fractal equation by Blackmore and Zhou [4, 5]. This equation generates a polar coordinate system that consists of $r$ and $\theta$ in the $x$-$y$ plane and the fractal coordinate in the $z$ direction. The equation can be written as follows:

$$z = \Phi(r, \theta) := \alpha r^{-2} \sum_{n=1}^{\infty} b^{(3-\alpha) n} \sin(b^{n} r + 1) \cos(b^{n} \theta) + \sqrt{a^{2} - r^{2}} - \sqrt{a^{2} - r_{0}^{2}}$$

(1)

This equation comprises and connects a cylinder cymbal shape and fractal surface height features on one top surface. The parameters used in this equation and their significance are given in Table 1. As shown in [5], this equation describes an approximately self-affine surface of fractal (box) dimension $s$ ($2 < s < 3$), and it follows from the definition that the surface is invariants under rotations about the $z$-axis (cf. [6]).

2.2 Design Methodology. As shown in the following flow chart (Fig.1), the methodology of creating a fractal surface cymbal consists of 5 basic steps, and each step needs to use specific equations, software, hardware and skills.

3. Testing and results
3.1 Fractal Equation Based Calculations. The main component of this project is the fractal equation that essentially generates a fractal $z$ coordinate for a given $x$ and $y$ coordinate pair [4, 7] or in our case, a polar coordinate system that consists of $r$ and $\theta$ in the $x$-$y$ plane and the fractal coordinate in the $z$ direction, see Equation (1). This equation and a Matlab program are used to create a series of 3D coordinates that are output into an ASCII file. In Equation (1) we aim to study the effect of the modification of the values of $\alpha$, $\beta$ and $s$ on the fractal profile of the cymbal.

3.1.1 Fractal Surface Measurement. While we vary the parameters, it is necessary to decide exactly what output we are measuring and using as a basis to decide the optimum values of ‘$\alpha$’ and ‘$\beta$’. The most common ways to measure a surface’s roughness or irregularity is the Arithmetic Mean Value ($R_a$) or the Root Mean Square Value ($R_q$) of surface roughness [9]. These indices are calculated by their respective formulae as follows:
Table 1. The various parameters and their characteristics.

<table>
<thead>
<tr>
<th>Parameter Symbol</th>
<th>Parameter Name</th>
<th>Units</th>
<th>Selected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>‘z’ axis coordinate</td>
<td>mm</td>
<td>Dependent</td>
</tr>
<tr>
<td>r</td>
<td>Radial coordinate</td>
<td>mm</td>
<td>Dependent</td>
</tr>
<tr>
<td>θ</td>
<td>Angular coordinate</td>
<td>Radian</td>
<td>0 – 2Π</td>
</tr>
<tr>
<td>α</td>
<td>Topothesy</td>
<td>Unit less</td>
<td>≥1</td>
</tr>
<tr>
<td>β</td>
<td>Constant integer</td>
<td>Unit less</td>
<td>5-10</td>
</tr>
<tr>
<td>s</td>
<td>Fractal surface dimension</td>
<td>Unit less</td>
<td>2&lt;s&lt;3</td>
</tr>
<tr>
<td>n</td>
<td>No. of iterations</td>
<td>Unit less</td>
<td>Approximately 10</td>
</tr>
<tr>
<td>r_o</td>
<td>Radius of Cymbal</td>
<td>mm</td>
<td>75 mm</td>
</tr>
<tr>
<td>a</td>
<td>Radius of curvature of cymbal</td>
<td>mm</td>
<td>4000 mm</td>
</tr>
</tbody>
</table>

\[
R_a = \frac{1}{l} \int_0^l |z| dz = z_1 + z_2 + z_3 + \ldots z_n \ldots \ldots \ldots (2) \\
R_q = \left[ \frac{1}{l} \int_0^l z^2 dz \right]^{1/2} = \left[ \frac{z_1^2 + z_2^2 + z_3^2 + \ldots z_n^2}{n} \right]^{1/2} \ldots \ldots (3)
\]

where \( z_1, z_2, \ldots \) are the \( z \) coordinates or the fractal values of the respective points in any two-dimensional cross-sectional plane and \( l \) is the length of the cross-sectional plane which is related to the cymbal radius. Given that our cymbal is a 3-dimensional object, it might prove more effective and accurate to measure roughness values \( R_a, R_q \) in one radial cross-sectional plane for all cross-sectional planes and then take their average value as our measurable output.

3.1.2 Fractal Dimension. The main deliverable outcome in this paper is a set of protocols and an understanding of the effect of the variation of the above mentioned parameters on the desirable fractal characteristics of the cymbal. By varying the value of the fractal parameter versus the surface roughness values for different values of ‘\( \alpha \)’ (topothesy) we obtain the following plots. As we can see from the figures 2 and 3, the roughness of the surface or the broken nature increases exponentially with an increase in the fractal parameter’s’. Successive increments of the topothesy (‘\( \alpha \)’) also create an increase in the roughness values, but at a more linear rather than exponential rate.

3.1.3 Relationship between other fractal parameters (‘\( \beta \)’) and surface roughness. The variation of the beta values does not cause any relative change in the roughness as compared to the topothesy or the fractal parameter values (Fig 4).

3.2 From Points to Surface Model. After some research, we were able to use software called Imageware to read 3D point coordinates from an ASCII file. This allowed us to create a small test surface of a square with about 150 points, Fig. 5 compared to our final goal of about 100,000 points. The next challenge in this project was to understand and use the surfacing parameters of Imageware. Initially we created a surface by using the uniform surface option that produces a smooth surface by averaging the point coordinates. As we progressed, we increased the number of points and improved the resolution greatly by adjusting the tension, smoothness, surface order and standard deviation of the surface. Another improvement in the process came when the initial Cartesian fractal equation was modified into a polar form which was better suited for a cymbal. The final step in creating the best possible surface with the given parameters came when the uniform surfacing tool was replaced with the surface interpolation tool in Imageware. In order to improve the accuracy, the interpolating surface option was used as a substitute; this tool created the surface by directly interpolating splines through each point based on the order of the surface, thereby introducing virtually zero error, see Fig. 6 (a)&(b). The final surface that was developed had about 120,000 points and a near zero error value with respect to the deviation between the surface and the points. Due to processing limitations of Imageware, Matlab and Pro/Engineer we have had to limit the number of points to about 120,000 for a cymbal of diameter 75mm. From the fractal cymbal equation, the generated point cloud is read and plotted by surfacing software Imageware which plots them in 3D space. These points are then prepared for surfacing by interpolating a
number of Bezier curves; the accuracy of the surface depends upon the order of the curves and the number or points per unit area of the surface. Fig. 7 (a) shows the initially read point cloud and (b) shows the processed interpolated point cloud, as seen in Imageware.

Fig. 2. Plot of ‘s’ versus ‘Rq’ for different ‘α’.

Fig. 3. Plot of ‘s’ versus ‘Ra’ for different ‘α’.

Fig. 4. Effect of Beta variation versus roughness.

Fig. 5. Initial test surface used to develop Imageware surfacing.

Fig. 6 (a) Uniform surface wireframe; b) Interpolated surface wireframe; (b) Interpolated point cloud.

Fig. 7 (a) Point cloud as seen in Imageware; (b) Interpolated point cloud.

3.3 From Surface Modeling to Solid Modeling. Once the interpolated point cloud has been created, it is possible to create a surface model from the interpolated curves in Imageware. By saving the surface model as a universal IGES file, solid modeling software such as PRO/E can open the surface model and then solidify this surface into a solid model as in Fig. 8. Saving this in a rapid prototype machine default code STL file [10, 11], the solid cymbal can be simulated in RP processing software and the result is shown in Fig. 9. The final rapid prototyped cymbal using our RP machine is shown in Fig. 10.

3.4 CNC Manufacturing. The final step in this process is the manufacture of the Cymbal for research and testing using CNC simulation software such as Mastercam. Fig. 11 (a) shows CAM software simulation of a tool path for cutting the cymbal, and (b) is a milling machine simulation of the cutting process.

4. Conclusion

The results we have obtained have shown the variation of the surface roughness with respect to equation parameters such as fractal dimension (‘s’), Topothesy (‘α’) etc. These results allow us to accurately predict the roughness of the surface for a given set of parameters. Also the representation of other complex higher order surfaces gives this equation based approach an additional emphasis for surface modeling. The equation based design
methodology can be used to create many complex surfaces that could not otherwise be created by conventional CAD methods. Also by graphing the variation of important characteristics such as roughness, curvature etc as the parameters of the equation are changed, we can get a good understanding of the basics of controlling the nature of the surface. The Final step in this process is the CAM (Fig.13 a & b) or prototyping (Fig.9 &10) of the solidified surface in order to carry out further tests on it. This post-manufacture testing is an important step to further characterize and understand the variation of the surface with the variation of the equation parameters.

References