Optical vortex metrology

Mitsuo Takeda\textsuperscript{a}, Wei Wang\textsuperscript{b}, Steen G. Hanson\textsuperscript{c}, Yoko Miyamoto\textsuperscript{a}

\textsuperscript{a}Department of Information and Communication Engineering, The University of Electro-Communications, 1-5-1, Chofugaoka, Chofu, Tokyo, 182-8585, Japan
\textsuperscript{b}Department of Mechanical Engineering, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom
\textsuperscript{c}Department of Photonics Engineering, Technical University of Denmark, Fotonik, P.O. Box 49, DK-4000 Roskilde, Denmark

Tel.: +81 [42] 443-5276 Fax: +81 [42] 489-6072 E-mail: takeda@ice.uec.ac.jp

Abstract

We review the principle and the applications of a new technique for displacement and flow measurements. The technique is called optical vortex metrology because it makes use of phase singularities in the complex signal as markers or tracers, which are generated by a vortex filter that performs a Riesz or Laguerre-Gauss transform operation to a speckle-like random pattern.

Keywords: Phase singularity, optical vortex, speckle, optical metrology, speckle photography

1. Introduction

The basic principle of electronic speckle photography is to compare images of the speckle patterns before and after displacement of an object based on the cross-correlation function of speckle intensity patterns, and no attention has been paid to the phase information associated with the speckle patterns. Recently, we proposed a new technique, called optical vortex metrology, which makes use of the phase singularities in the pseudophase of the complex signal obtained from the Riesz transform or the Laguerre-Gauss transform of speckle patterns [1, 2]. Although this complex representation of real-valued speckle patterns does not introduce new information, it effectively exploits the existing information in such a manner that the newly introduced pseudoamplitude and pseudophase associated with the complex signal provide a powerful means for analyzing, processing and understanding the available information from the recorded speckle pattern. Furthermore, because the pseudophase can be detected without recourse to interferometry and the principle is based on tracing individual phase singularities as displacement markers, the proposed technique has the versatility that expands applications beyond those known for conventional correlation-based laser speckle metrology. In this paper, we review the principle and some of the applications of optical vortex metrology.

2. Principle

It is common practice in physics and engineering to represent real-valued signals by the related complex-valued signals. For one-dimensional (1-D) signals, the concept of analytic signals based on the partial Hilbert transform was introduced to communication theory by Gabor in the 1940s. To obtain an isotropic 2-D analytic signal for a speckle pattern, we replace the partial Hilbert transform with a vortex transform, such as the Riesz transform [3] and the Laguerre-Gauss (LG) transform [4]. While the Riesz transform is implemented by an all-pass spiral pure-phase filter in the spatial frequency domain, the LG transform is implemented by a band-pass filter with a spiral phase, which has a freedom to tune the band-pass characteristics to obtain stable and optimally distributed phase singularities.
Let $g(x, y)$ be the original intensity distribution of a speckle-like pattern, and let its Fourier spectrum be $G(f_x, f_y)$. The isotropic 2-D complex signal $\tilde{g}(x, y)$ is generated by

$$\tilde{g}(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V(f_x, f_y) G(f_x, f_y) \exp\left[j2\pi(f_x x + f_y y)\right] df_x df_y ,$$

where the vortex filter is given by

$$V(f_x, f_y) = (f_x + j f_y) / \sqrt{f_x^2 + f_y^2} = \exp\left[j\beta(f_x, f_y)\right] ,$$

for the Riesz transform, and

$$V(f_x, f_y) = \sqrt{f_x^2 + f_y^2} \exp\left[-\left(f_x^2 + f_y^2\right) / w^2\right] \exp\left[j\beta(f_x, f_y)\right] ,$$

for the Laguerre-Gauss (LG) transform, with $\beta(f_x, f_y) = \arctan(f_x / f_y)$ being the spiral phase, and $w$ a filter parameter for controlling the spatial frequency pass band. In the signal domain Eq. (1) can be rewritten as

$$\tilde{g}(x, y) = |g(x, y)| \exp\left[j\theta(x, y)\right] = g(x, y) * v(x, y) ,$$

where $*$ is a convolution operation and the convolution kernel is given by

$$v(x, y) = j(x + jy) / \left[2\pi(x^2 + y^2)^{3/2}\right]$$

for the Riesz transform, and

$$v(x, y) = j\pi^2 w^2 (x + jy) \exp\left[-\pi^2 w^2 (x^2 + y^2)\right]$$

for the LG transform; these vortex kernels have a self-similarity in their forms with their Fourier transforms. The phase $\theta(x, y)$ of the complex signal of the speckle-like pattern is referred to as the pseudophase to distinguish it from the true phase of the optical field. Although it is not the true phase of the complex optical field, the pseudophase does provide useful information about the object.

Just as a random speckle intensity pattern imprints marks on a coherently illuminated object surface, randomly distributed phase singularities in the pseudophase map of the speckle-like pattern imprint unique marks on the object surface. We make use of the information about the locations of phase singularities before and after the displacement. To do this, we need to identify the corresponding phase singularities between the pre- and post-displacement phase maps. If the displacement is known to be small a priori, we can restrict our search only to the closest neighbor phase singularities of the same topological charge. However, when the displacement is large and/or non-uniform and no a priori information is given, we cannot uniquely identify the corresponding phase singularities. To solve this problem, we make use of additional information about the core structure of the phase singularity.

![Fig. 1. Core structure around a phase singularity with zero crossings of real and imaginary parts of complex signal representation for a speckle pattern. (a) Amplitude contours and zero crossing lines; (b) Pseudophase structure.](image-url)
singularities. Similarly to optical vortices in random laser speckle fields [5, 6], the changes of the pseudophase around the phase singularities are non-uniform, and the typical core structure around the phase singularities are strongly anisotropic. Figure 1 shows an example of the amplitude contours and neighborhood of a phase singularity; the phase has a characteristic feature of a $2\pi$ helical structure. Usually, the real and imaginary parts of the complex signal representation of a 2-D speckle pattern in the immediate vicinity of a phase singularity can be approximated by the planes

\[
\Re[\hat{g}(x, y)] = a_x x + b_y y + c, \quad \Im[\hat{g}(x, y)] = a_x x + b_y y + c_i
\]

where the parameters $a$, $b$, and $c$ can be determined by least-square fitting the planes to the detected complex values at the pixel grids surrounding the phase singularity [1]. The phase singularity is located at the center of the elliptical contours of the amplitude, which is the intersection of the zero crossings of the real and imaginary parts of the complex signal $\hat{g}(x, y)$. We note that the eccentricity of the contour ellipse $e$ and the zero crossing angle $\theta_{RI}$ between the real and imaginary parts, shown in Fig. 1(a), are invariant to the in-plane rigid-body motion of the object involving translation and rotation, and we use these two geometric parameters to describe the local properties of the phase singularities. In addition, each phase singularity has its own topological charge and vorticity defined by $\Omega = \nabla \times \nabla \{\Re[\hat{g}(x, y)]\} \times \nabla \{\Im[\hat{g}(x, y)]\}$, which we also assume to be invariant to the in-plane rigid-body displacement involving translation and rotation. Just as no fingers have exactly the same fingerprint patterns, no phase singularities have exactly the same local properties with identical eccentricity $e$, zero-crossing angle $\theta_{RI}$, topological charge $q$, and vorticity $\Omega$. It is this uniqueness of the core structure that enables the correct identification and the tracking of the complicated movements of phase singularities. The matching between the pre- and post-displacement phase singularities are done on the basis of a merit function representing the degree of similarity between these parameters expressed by a distance in their parameter space.

3. Experiments

To measure lateral displacements in the micrometer range and rotational micro-displacements, we introduced controllable lateral and rotational micro-displacements with the precision mechanical stage of the microscope. From the recorded random texture patterns on the surface of the precision stage, we generated an isotropic complex signal by Laguerre-Gauss filtering and retrieved the pseudophase information. In the experiment, we adjusted the average speckle size and controlled the density of phase singularities carefully by choosing a

![Fig. 2. Movement of phase singularities caused by (a) lateral micro-displacement, and (b) rotational micro-displacement.](image-url)
proper bandwidth of the LG filter $w$ in Eq. (3), so that a single speckle includes about 40 pixels along a traversing line. After identifying the corresponding phase singularities for the object before and after displacement, we measured the given displacement by the proposed optical vortex metrology. The details of the experiments are described in Ref. [2]. Figure 2 (a) shows the parallel movement of phase singularities caused by the lateral displacement introduced by the translation stage. The displacements in the $x$ and $y$ directions were found to be $(\Delta x, \Delta y) = (19.6 \pm 0.4 \mu m, 21.2 \pm 0.3 \mu m)$. Figure 2 (b) shows the rotational movement of phase singularities caused by the rotational displacement introduced by the rotation stage. The rotation angle was found to be $-518$ milli-radian with the standard deviation 2 milli-radian.

We have also demonstrated optical vortex metrology for displacement measurement with nanometric resolution [1], and for random flow measurements [7].

In addition to the information about the anisotropic core structure for an individual phase singularity as its unique fingerprint for reliable identification, we detected a group of optical vortices (singularity clusters) with specific mutual spatial structure to further strengthen an unambiguous tracking over the entire field of view [8]. To demonstrate the performance of the proposed technique, a swimming fugu fish with a speckle-like intensity pattern on its body surface, as seen in Fig. 3 (a) and (b), was used as a biological specimen.

![Fig. 3. Recorded images for the swimming fugu at different instants of time and the generated Laguerre-Gauss signals with the trajectories of pseudophase singularities inserted: (a) and (c) are recorded at $t=0.70$ seconds; (b) and (d) at $t=3.33$ seconds.](image)

First, a series of images of the swimming fugu fish were recorded by a high-speed camera, FASTCAM-NET 500/1000/Max (PHOTRON) with a pixel size of $7.4 \mu m \times 7.4 \mu m$. From the recorded images, we generated an isotropic complex analytic signal by LG filtering, and retrieved the pseudophase information. We adjusted the average speckle size by choosing a proper bandwidth of the LG filter in Eq. (3). In the experiment, we have controlled the density of the pseudophase singularities carefully so that the phase singularities within the fugu body are clearly separated with a large average separation distance. Then, the constellation of phase singularities was identified and tracked during the specimen’s motion across the entire field of view of the camera. After identifying corresponding pseudophase singularities for each pair of
consecutive images making use of their core structures as fingerprints, we traced the movement of the swimming fugu through its trajectory as shown in Fig. 3 (c) and (d), where the arrows indicate their movement directions at different instants of time. As expected for the rotation of fugu’s swimming, the trajectory exhibits an arch shape, and body part far from the gravity center of fugu shows a larger arc length. From the coordinate information for each phase singularity in the constellation, we conducted the in vivo measurement and obtain instant information for this swimming fugu about its translation, rotation, and scaling [8].

4. Conclusion
We reviewed a new technique of optical vortex metrology for translational and rotational displacement measurements that makes use of the pseudophase singularities in the complex signal representation of a speckle pattern generated by LG filtering. As indicators of local spackle displacement, these pseudophase singularities can be traced through their core structures, and the displacement of an object can be estimated from the subsequent location displacement of the registered phase singularities. The singularity constellation uniquely characterizes the mutual positional relation between the individual phase singularities, and can be used for the purpose of unique identification of a cluster of pseudophase singularities. We introduced experiments that demonstrate the validity of the proposed technique for non-uniform displacement measurements.

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References