Spatial phase-shifting moiré tomography

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Abstract

A novel spatial phase-shifting method using moiré phenomena is presented. According to our analysis of moiré patterns on the base of scalar diffraction theory there is stable phase shift between fringe patterns of different order filtering. This phase shift is the strict value of $+\beta$ and $-\beta$ and just depend on the distance between two gratings and the period of gratings. In this spatial phase-shifting method, no beam splitter, polarizer or wave plate are used and the disadvantage of uncertain phase shifts due to the methods using polarizer or wave plate is removed.

Keywords: Tomography, tomographic image processing, moiré techniques, spatial phase-shifting, diffraction theory

1. Introduction

There is an increasing need for real-time visualization and quantitative measurement of phase objects. Moiré deflectometry is an efficient and important tool in these fields, such as optical instruments test, shock wave measurements in wind tunnels and temperature detections in flame flow fields. Compared with interferometer, moiré deflectometry has advantages of low mechanical stability demand and high dynamic range. These advantages are particularly important in noisy environments. Several fringes processing methods, included the fringe detection method [1] based upon the image intensity, the electronic heterodyne method [2], the Fourier transform method [3] and the temporal phase-shifting method [4] have been applied to extract deflection projections from moiré fringes in moiré tomography. However, with the temporal phase-shifting method, the measured data are vulnerable to disturbances during the phase-shifted images recordings. The spatial phase-shifting methods can achieve phase-shifted images simultaneously and are capable of solving this problem.

The spatial phase-shifting methods have been widely used in the optical profilometry to characterize an object surface. The common methods to achieve several phase-shifted images use beam splitters [5, 6], gratings [7, 8] to divide the probe light and use wave-plates [5, 6, 8] or polarizer [7] to add phase-shifting modulation. Isaac Amidror [9] has studied the moiré phenomena with spectral approaches and found that the phase shifting occurs when one or more grating is shifted to a small period.

In this paper, we present a novel spatial phase-shifting method using moiré phenomena. No polarizer, wave plate or beam-splitter is necessary and it is no necessary to shift any gratings. We achieve the formation of moiré fringes from the scalar diffraction theory and find there is a stable phase shifting between the intensity distributions of first order filtering, zero order filtering and negative-one order filtering. This phase shifting only depends on the period of gratings and the distance between these two gratings in moiré deflectometry.
2. Theoretical analysis

A. Moiré patterns from the scalar diffraction theory

Figure 1 is the optical schematic diagram of the moiré deflectometry. G1 and G2 are two identical ronchi gratings that are illuminated with collimated monochromatic coherent light. They are set a distance $\Delta$ apart and oriented at angles $\alpha/2$ and $-\alpha/2$, respectively, relative to the y axis. $f$ is the focal length of lens L1 and L2. F is a pinhole filter. OP is the plane of observation. So these lens L1, L2 and filter F make up a 4-f system. Figure 2 shows that there is a shift $b$ between grating G1 and G2.

When a refracting object O is placed before the grating G1, the incident plane wave before G1 is distorted. Because the phase distribution of the distorted wave front is actually the phase projection $\varphi(x, y)$, the field before G1 is

$$u_i^-(x, y) \propto \exp[ik\varphi(x, y)]$$

(1)

Where $k = 2\pi/\lambda$ and $\lambda$ is the wavelength. The grating G1 forms an angle of $+\alpha/2$ with respect to the y axis and is considered infinite in extent. $u_i^+(x, y)$ is the field behind the grating G1. Let $(u, v)$ is the Fourier spatial frequency components. So the corresponding angular spectrum $U^+_1(u, v)$ is

$$U^+_1(u, v) = \sum_{m} a_m U_i^- (u - \frac{m}{d} \cos \frac{\alpha}{2}, v + \frac{m}{d} \sin \frac{\alpha}{2})$$

(2)

d is the period of gratings. With the concept of the angular spectrum propagation, the angular spectrum $U^-_2(u, v)$ for the plane just preceding G2 is

$$U^-_2(u, v) = \exp[ik\Delta \sqrt{1-\lambda^2(u^2+v^2)}] \sum_{m} a_m U_i^- (u - \frac{m}{d} \cos \frac{\alpha}{2}, v + \frac{m}{d} \sin \frac{\alpha}{2})$$

(3)

The grating G2 forms an angle of $-\alpha/2$ with respect to the y axis and have a shift $b$ respect to grating G1. The angular spectrum of the field $u_j^+(x, y)$ behind the grating G2 is
\[ U_2^*(u, v) = \sum_{(m, n)} a_m a_n \exp \left( -i \frac{2\pi nb}{d} \right) U_1^* \left( u \frac{m+n}{d} \cos \frac{\alpha}{2}, v \frac{m+n}{d} \sin \frac{\alpha}{2} \right) \]
\[
\times \exp \left[ ik\Delta \sqrt{1 - \Delta^2 \left[ (u - \frac{n}{d} \cos \frac{\alpha}{2})^2 + (v - \frac{n}{d} \sin \frac{\alpha}{2})^2 \right]} \right] \tag{4}
\]

The field \( u_2^*(x, y) \) behind the grating G2 can be achieved from the inverse Fourier transform.
\[
u_2^*(x, y) = \exp (ik\Delta) \sum_{(m, n)} a_m a_n \exp \left( -i \frac{2\pi nb}{d} \right) \exp \left[ i2\pi d \left( (m+n)x \cos \frac{\alpha}{2} - (m-n)y \sin \frac{\alpha}{2} \right) \right]
\times \exp \left( -i \frac{\pi \lambda \Delta m^2}{d^2} \right) \mu_i \left( x - \frac{\lambda \Delta m}{d} \cos \frac{\alpha}{2}, y + \frac{\lambda \Delta m}{d} \sin \frac{\alpha}{2} \right) \tag{5}\]

Performing the Taylor series expansion to \( \varphi(x, y) \). Substituting the first two terms into Eq. (5)
\[
u_2^*(x, y) = \exp (ik\Delta) \sum_{(m, n)} a_m a_n \exp \left( -i \frac{2\pi nb}{d} \right) \exp \left[ i2\pi d \left( (m+n)x \cos \frac{\alpha}{2} - (m-n)y \sin \frac{\alpha}{2} \right) \right]
\times \exp \left( -i \frac{\pi \lambda \Delta m^2}{d^2} \right) \exp \left[ ik \varphi(x, y) \right] \exp \left( -i \frac{\partial \varphi(x, y)}{\partial x} \frac{\lambda \Delta m}{d} \cos \frac{\alpha}{2} \right) \tag{6}\]

Because of the 4-f system which consists of lens L1, L2, the field \( u(x, y) \) on the plane of observation should be the same as \( u_2^*(x, y) \). Figure 3 shows the spectrum distribution on the filter plane which locates one focal length behind the lens L1. The distribution of frequency spectrums has a diamond structure with a large angle \( \alpha \). While decreasing the angle, they combine to be a line. The \( p \)th order frequency spectrum in Fig. 3(b) can be regard as the multiple-shearing interference of frequency spectrums which satisfy \( n + m = p \). As shown in Fig. 3, the diffractive energy mainly focuses on frequency spectrums which satisfy \( m + n = (-2, -1, 0, +1, +2) \) and other frequency spectrums can be ignored.

\[
m+n= \begin{array}{cccccccc}
\ldots & +2 & +1 & 0 & -1 & -2 & \ldots \\
\end{array}
\]

(a)

(b)

Fig. 3. The spectrum distribution of \( u_2^*(x, y) \). (a) \( \alpha \gg 0 \). (b) \( \alpha \rightarrow 0 \).
B. Zero-order filtering

The multiple-shearing interference can be simplified by spatial filtering. Our first consideration is the case of zero-order, \( n+m=0 \). As shown in Fig. 3, significant contributions to the zero order comes from three frequency spectrums including \((m=0, n=0), (m=1, n=-1)\) and \((m=-1, n=1)\). The field with the zero-order filtering can be considered a triple-shearing interference. The intensity distribution on the plane of observation is

\[
I_0(x, y) = a_0^4 + 4a_0^2a_1^2 \cos \left( \frac{\pi \Delta}{d^2} \right) \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} - \frac{2\pi b}{d} \right) \\
+ 4a_1^4 \cos^2 \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} - \frac{2\pi b}{d} \right)
\]

(7)

C. First-order filtering

To perform the first-order filtering, we select either the \( m+n=1 \) or \( m+n=-1 \) term. Substituting \( m+n=1 \) into Eq. (11) and taking two major frequency spectrums ((\( n=0, m=1 \)), \( (n=1, m=0) \)). The intensity distribution in the plane of observation is

\[
I_+ (x, y) = 2a_0^2a_1^2 \left[ 1 + \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} + \pi \frac{\Delta}{d^2} - \frac{2\pi b}{d} \right) \right]
\]

(8)

Substituting \( m+n=-1 \) into Eq. (11) and the intensity distribution with -1 order filtering is

\[
I_- (x, y) = 2a_0^2a_1^2 \left[ 1 + \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} + \pi \frac{\Delta}{d^2} + \frac{2\pi b}{d} \right) \right]
\]

(9)

3. The phase shift between fringes

Compared the above Eq. (8) and Eq. (9), we can find a phase shift term \( \pi \Delta/d^2 \). This phase shift term just depend on the distance \( \Delta \) between two gratings and the period \( d \) of gratings. If \( \Delta \) and \( d \) is defined, the phase shift of the +1 order and -1 order filtering is \( \pi \Delta/d^2 \) and \( -\pi \Delta/d^2 \) respect to the zero order filtering. If the distance \( \Delta \) is the Talbot distance, \( \Delta = Kd^2/\lambda \). The results of +1 order, -1 order and zero order filtering are:

\[
I_+ (x, y) = 2a_0^2a_1^2 \left[ 1 + \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi Kd}{\lambda} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} + \pi K - \frac{2\pi b}{d} \right) \right]
\]

(10)

\[
I_0 (x, y) = \left[ a_0^4 + 2a_0^2a_1^2 \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi Kd}{\lambda} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} - \frac{2\pi b}{d} \right) \right]^2
\]

(11)

\[
I_- (x, y) = 2a_0^2a_1^2 \left[ 1 + \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi Kd}{\lambda} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} - \pi K - \frac{2\pi b}{d} \right) \right]
\]

(12)

In this case, the fringe pattern with the zero order filtering have the best contrast. The phase shift between three patterns is \( \pi, 0 \) and \( -\pi \). We can’t get useful phase shifting information. If the distance \( \Delta \) is the Sub Talbot distance, \( \Delta = (2K+(1/2))(d^2/\lambda) \). The results of +1 order, -1 order and zero order filtering are:

\[
I_+ (x, y) = 2a_0^2a_1^2 \left[ 1 + \cos \left( \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4\pi}{d} y \sin \frac{\alpha}{2} + \frac{\pi}{2} - \frac{2\pi b}{d} \right) \right]
\]

(13)
\[
I_0(x, y) = a_0^4 + 4a_1^4 \cos^2 \left[ \frac{\partial \varphi(x, y)}{\partial x} \frac{2 \pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4 \pi}{d} y \sin \frac{\alpha}{2} - \frac{2 \pi b}{d} \right]
\] (14)

\[
I_{-1}(x, y) = 2a_0^2 a_1^2 \left\{ 1 + \cos \left[ \frac{\partial \varphi(x, y)}{\partial x} \frac{2 \pi \Delta}{d} \cos \frac{\alpha}{2} + \frac{4 \pi}{d} y \sin \frac{\alpha}{2} - \frac{\pi}{2} - \frac{2 \pi b}{d} \right] \right\}
\] (15)

In this case, the fringe pattern with the zero order filtering have the worst contrast. The phase shift between three patterns is \( \pi/2 \), 0 and \(-\pi/2\).

![Fig. 4. Moiré patterns with the sub Talbot distance (a) +1 order filtering (b) -1 order filtering.](image)

![Fig. 5. Moiré patterns with zero order filtering (a) the Talbot distance (b) the sub Talbot distance.](image)

The moiré patterns of propane flame with 532 nm laser are shown as Fig. 4 and Fig. 5. Figure 4 shows the moiré patterns of +1 order filtering and -1 order filtering at the sub Talbot distance, respectively. The white cross indicates the origin of each image. It is easy to find a \( \pi \) phase shifting between these two images. Figure 5 shows the moiré patterns of zero order filtering at the Talbot distance and the sub Talbot distance, respectively. Figure 5(a) have the best contrast ad the origin which is indicated by the white cross locates upon the border of fringes. So from Fig. 4 and Fig. 5 (a), we can find the phase shifts \( \pi/2 \), \(-\pi/2\), 0 respect to each images. Figure 5 (b) shows the worst contrast with the zero filtering.

The two-step phase-shifting algorithm [10] can be applied to extract the phase term \( \phi(x, y) \) from the +1 order filtering and the -1 order filtering.
\[
\phi(x, y) = \frac{\partial \phi(x, y)}{\partial x} \frac{2\pi \Delta}{d} \cos \alpha \frac{\alpha}{2}
\]

(16)

So the projection data of deflection angles can be achieved by [11]

\[
\varphi_d(x, y) = \frac{d}{2\pi \Delta} \phi(x, y) \left( -\frac{1}{n_0 \cos(\alpha/2)} \right)
\]

(17)

Where \( n_0 \) is the ambient refractive index.

In practice, it is not necessary to limit the distance \( \Delta \) to be a pre-defined value, such as the sub Talbot distance. The distance can be any value except the talbot distance. The phase shift \( \beta \) is

\[
\beta = \frac{\pi \lambda \Delta}{d^2}
\]

(18)

The shift \( b \) between do not take any phase shift between images because there are the same factor \( -2\pi b/d \) in all intensity equations. If the grating \( G1 \) is shifted, there is a phase shifting from the origin image. This result is the same with Isaac Amidror’s [9]. This can be applied to the temporal phase-shifting methods. However, in the spatial phase-shifting methods, the shift \( b \) is unimportant.

References