Research on the vibration resistance ability of the random phase-shifting interferometry

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Abstract
The Phase-shifting Interferometry has been widely used because of its high measurement accuracy and high spatial resolution. However, environmental vibration, air turbulence or inaccurate phase shift are inevitable in the working field and may cause measurement errors. A novel Random Phase-Shifting Interferometry (RPSI) is proposed in this paper. Large numbers of phase-shifting interferograms are gathered and analyzed, and a phase front independent from vibration and air turbulence can be reconstructed. Also the requirement on phase-shifting device is relaxed in RPSI. In this paper, first the measurement principle of RPSI is described in details and then the vibration resistance ability of this method is further theoretically simulated. The relationship between measurement accuracy and environmental vibration is obtained. Finally, experiments with a Fizeau interferometer were carried out to demonstrate the feasibility and performance of RPSI.

Keywords: Random phase-shifting, vibration resistance, interferometry

1. Introduction
Phase-shifting interferometry has been widely used in surface testing of optical elements. Due to the large size of the measured surface, the optical path length is set to be several meters or even longer to complete the testing. As a result vibration and air turbulence will change the preset phase shifts and cause inevitable errors to the measurement [1].

Vibration-insensitive phase-shifting interferometry has been developed over the years. Phase feedback methods [2] with mechanical phase-shifting or acousto-optic modulation can compensate vibrations of small amplitude but suffer from low contrast interferograms. One-shot or simultaneous phase-shifting interferometers [3] succeeded in reducing time-varying noises by capturing the interferograms at the same time with different parts of the same detector. But complicated optical elements are needed to introduce the phase shifts and the spatial resolution will be limited. An anti-vibration phase-shifting interferometry [4] was developed to use vibration as the only phase-shifter to measure large-aperture mirrors. This method is accurate but very time-consuming because the vibration is uncontrollable.

In this paper, Random Phase-Shifting Interferometry is proposed. The relationship between intensity and phase in each pixel is obtained from large amount of phase-shifting interferograms. Averaging statistically uncorrelated data over a long enough period of time can effectively reduce most random errors. RPSI can be applied to all kinds of surface measurement interferometers. The vibration resistance ability of it is worth further research.
2. Measurement principle

Different from most of the other phase-shifting interferometry, in the RPSI phase-solving algorithm, the phase shift step length is not taken as a parameter, but the temporal intensity maximum and minimum in each pixel are needed. For finding the extreme values, random phase shifts caused by environmental noises are adopted to make the intensity ergodic. Supplementary active phase shifts, which are not accurately controlled or calibrated, are introduced to shorten the measurement cycle.

Fig. 1. Sketch of the intensity curve against the frame index.

Suppose $N$ frames of interferograms are collected and the total phase shift is over one or two periods. The intensity variation of an arbitrary pixel $(x, y)$ can be plotted against the frame index as in Fig. 1, and the intensity (grayscale) in the $i$th interferogram can be expressed as

$$I(x, y, i) = I_b(x, y) + I_a(x, y) \cdot \cos[\Phi(x, y) + \delta(x, y, i)],$$

where $I_b(x, y)$ and $I_a(x, y)$ present the background intensity and modulation amplitude in pixel $(x, y)$ respectively; $\Phi(x, y)$ is the original phase in pixel $(x, y)$ which is the target of the measurement, and $\delta(x,y,i)$ is the effective phase shifts corresponding to the $i$th frame. The maximum and minimum intensity in each pixel, denoted as $I_{\text{max}}(x, y)$ and $I_{\text{min}}(x, y)$, can be found out from the measurement points, as they are marked out in Fig. 1. These extreme values are simply related to the background intensity and the modulated amplitude and then $I_b(x, y)$ and $I_a(x, y)$ in Eq. (1) can be derived from

$$\begin{align*}
I_b(x, y) &= \frac{I_{\text{max}}(x, y) + I_{\text{min}}(x, y)}{2} \\
I_a(x, y) &= \frac{I_{\text{max}}(x, y) - I_{\text{min}}(x, y)}{2}.
\end{align*}$$

With the background intensity and modulated amplitude already known, the total phase in Eq. (1), denoted as $\phi(x, y, i) = \Phi(x, y) + \delta(x, y, i)$, corresponding to pixel $(x, y)$ in the $i$th frame can be solved from the intensity with arc cosine function

$$\phi(x, y, i) = \arccos \left[ \frac{I(x, y, i) - I_b(x, y)}{I_a(x, y)} \right].$$

Eq. (3) is the phase solution of this method and $\phi(x,y,i)$ is the wrapped principal phase. After 2D phase unwrapping in the spatial-domain, wavefront distributions corresponding to different sampling time are ready. These wavefront distributions may be distorted because of air turbulence and other random factors in image capturing or phase calculation. Consequently, all the $N$ frames of wavefront distributions are averaged to get a final measurement result

$$\phi'(x, y) = \frac{\sum_{i=1}^{N} W \left[ \phi(x, y, i) \right]}{N},$$

where $W$ represents a weighting function.
where $W$ is a spatial unwrapping operator.

Next a brief analysis on $\varphi'(x, y)$ will be given to further look into the noise insensitive feature of RPSI. Since $\varphi(x, y, i)$ comprises three parts, the original phase $\Phi(x, y)$, active phase shifts $\delta_a(i)$ and random phase shifts $\delta_r(x, y, i)$, the final result $\varphi'(x, y)$ will reflect these three contributors. Although $\delta_a(i)$ is not accurately controlled or measured, it should be the same for all the pixels in the aperture or namely it should be a piston phase shift. Mechanical vibration phase shift in $\delta_r(x, y, i)$ is also a random piston phase shift. Air turbulence phase shift and surface tilt error phase shift in $\delta_r(x, y, i)$ will change the phase randomly and locally. After the average phase calculation over large amount of interferograms in Eq. (4), the active phase shift and vibration phase shift will be a constant for all the pixels. And the random phase shift caused by air turbulence and surface tilt approximates zero. The final result $\varphi'(x, y)$ has the same shape with the original phase $\Phi(x, y)$, only with the average piston phase shift as a bias over the whole aperture. So theoretically this method is insensitive to the air turbulence and other local random errors.

3. Vibration resistance ability simulation

In order to verify the feasibility and accuracy of RPSI, a simulation of measuring a plane wavefront was done under the assumption that vibration exist in the measurement. Firstly a series of ideal measured wavefront are expressed with the ideal active phase shifts $\delta_a(i)$ as a parameter. Then vibration is introduced to the system and piston phase shifts to the whole wavefront are caused. The effective phase shift can be expressed as

$$\delta(x, y, i) = \delta_a(i) + \sum_{j=1}^{m} A_j \cdot \sin(2\pi f_j t + \alpha_j), \quad (5)$$

where $A_j$, $f_j$ and $\alpha_j$ are the amplitude, frequency and original phase of the vibration respectively; here the vibration can be decomposed into $m$ frequency components. Finally the series of modified wavefront interfere with a reference wavefront and large amount of interferograms are produced. The phase solution and unwrapping method present in Part 2 is used to solve the wavefront from the interferograms. Since the measured wavefront is a perfect plane wave, the peak-valley (PV) value of the result wavefront is taken as the measurement error.

Vibration is mainly described with the amplitude $A_j$ and the frequency $f_j$. Actually the vibration should be applied to the measured surface and the piston aberration of the whole measured wavefront is the consequence of it. Therefore $A_j$ is measured with the wavelength as the unit. For example, amplitude of $\lambda$ means that the vibration will bring in a maximum piston wavefront aberration of $\lambda$ (i.e., a phase shift of $2\pi$). The vibration frequency should be compared with the active phase-shifting speed and the sampling frame frequency of the image capture system. Here the ideal active phase-shifting speed is set to be $\lambda$ per second and the sampling frame frequency is 100 fps.

Simulations are done with different vibration amplitude and frequency. For the sake of clarity, the vibration has a single frequency component. If the vibration frequency is fixed and the amplitude is gradually increased, the measurement error will also increase and dramatically grows as shown in Fig. 2. In fact, when the measurement error “jumps” to greater than 0.1$\lambda$ in Fig. 2, there may be some information miss or wrong unwrapping area in the result wavefront. It is a symbol of measurement failure. In this situation, the vibration
frequency and amplitude corresponding to the accuracy of 0.1λ are defined as the critical frequency and amplitude for that accuracy.

![Graphs showing simulated measurement accuracy and different vibration amplitude and frequency](image)

Fig. 2. Simulated measurement accuracy and different vibration amplitude and frequency.

From careful observation of Fig. 2, it is found that the critical frequency and amplitude are 1.1 Hz / 0.500λ and 3.5 Hz / 0.148λ, and the products of these two parameters are close to each other (0.550 and 0.518). Similar comparison shows that the vibration critical amplitude-frequency product has a steady relationship with the measurement accuracy so that the product can be regarded as the vibration resistance quota. Some simulation results are shown in Table 1. The amplitude-frequency product gets greater when the accuracy is lower, which coincides with the physical intuition.

Table 1. Simulated measurement accuracy and vibration Amplitude-Frequency Product (AFP).

<table>
<thead>
<tr>
<th>PV (λ)</th>
<th>1/100</th>
<th>1/90</th>
<th>1/80</th>
<th>1/70</th>
<th>1/60</th>
<th>1/50</th>
<th>1/40</th>
<th>1/30</th>
<th>1/20</th>
<th>1/10</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFP (λ Hz)</td>
<td>0.3529</td>
<td>0.3622</td>
<td>0.3752</td>
<td>0.3889</td>
<td>0.4047</td>
<td>0.4182</td>
<td>0.4369</td>
<td>0.4676</td>
<td>0.5054</td>
<td>0.5586</td>
</tr>
</tbody>
</table>

As a conclusion for the simulation, RPIS shows good performance when the vibration frequency is low and the amplitude is small. This is a valuable progress because most of the phase-shifting interferometry methods do not work well in low-frequency vibration environments. Moreover, the vibration amplitude-frequency product determines the measurement accuracy when the system parameters, such as active phase-shifting speed or sampling frame frequency, are fixed. Further simulations indicate that the measurement accuracy can be improved when the active phase-shifting speed and the sampling frame frequency are increased in proportion.

4. Experimental demonstration

A Fizeau interferometer shown in Fig. 3 was built up to demonstrate this method. The Fizeau layout was adopted to save space and reduce errors with common-path settings. The standard plane SP as well as the image capture system was included in the interferometer (shown inside the dash frame). CCD1 camera was used for adjustment of the interferometer and CCD2 records the interferograms. The whole system worked on a common laboratory desk without special vibration isolation or air flow control measures. The active phase shifts were realized with a piezo-electric transducer PZT pushing the workpiece WP continuously in
a certain direction and the total active phase shift during the sampling time was about $2\pi$.

A plane as WP with an aperture of 15 mm was measured with this system. 100 frames of interferograms were captured at a frame frequency of 10 fps. The final wavefront reconstructed from the measurement result is shown in Fig. 4, with a PV value of $0.027\lambda$ where $\lambda = 632.8$ nm. As for the intermediate wavefront reconstructed from only one piece of interferogram, the PV value could be as high as $0.056\lambda$. This is a good proof that the average of many wavefront distributions indeed reduces random errors. The same plane was tested with a Zygo phase-shifting interferometer placed on a vibration isolator in an ultra-clean laboratory. Four-step phase-shifting method was used and the PV value of the wavefront result was $0.022\lambda$. In fact, the results from Zygo interferometer were temporally varying because of random factors. For example, the PV value ranged from $0.020\lambda$-$0.025\lambda$ in five times of measurements. So the result from RPSI coincided well with that from Zygo interferometer and the interferometer with RPSI has the advantages of low cost and relaxed requirements on the phase-shifting accuracy and the environment.

5. Conclusion

In this paper, a novel Random Phase-Shifting Interferometry is proposed. The detailed measurement principle and simulation of the vibration resistance ability are given. Theoretically RPSI is insensitive to air turbulence and other random noises. Moreover, RPSI may work well in low-frequency vibration environment and the amplitude-frequency product of the vibration can be taken as the vibration resistance quota. The feasibility and accuracy of RPSI was experimentally demonstrated with a Fizeau interferometer.

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References