Nanoscale measurement technique of in-plane motion for MEMS based on correlation fitting calibration method

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Abstract
Phase correlation method has also been extended into sub-pixel accuracy with varying levels of success. In this paper a completely novel calibration algorithm based on phase correlation method is derived. The theoretical limits of displacement sensing and estimation is improved by presenting calibration models for idealized algorithms. MEMS motion process is analyzed using the algorithm, which estimates MEMS displacements to within a few nanometers with optical microscopy. The offered solution achieves nanoscale accuracy. The amplitude-frequency curve is acquired through sweep frequency measurement. Experimental results indicate that the resolving power of measurement is less than 5nm.

Keywords: MEMS resonator, dynamic characteristic, phase correlation, subpixel

1. Introduction
Advanced testing methods for the dynamics of micro-devices are necessary to develop reliable marketable micro electro mechanical systems (MEMS). The main purpose for MEMS testing is to provide feedback to the design-and-simulation process in an engineering development effort. Motion measurement is a key method to get characteristic and dynamic parameters of MEMS resonator in every moment. The results of measurement give important reference to MEMS designation [1]. With the superiority of non-contact, high accuracy and full-field measurement, optical measurement methods are clearly well suited for MEMS characterization, and have been widely applied to the measurement of microstructures’ profile and motions [2, 3].

In this paper, we present a suite of algorithms, which provide image based displacement sensing and estimation. The algorithms are organized into two related families: The first family of algorithms is a compilation of Phase correlation algorithms. Phase correlation algorithms model a displacement as a constant phase delay across all the frequencies, which comprise the target system, and extract this phase delay by observing all available frequencies before and after the displacement. The second family of algorithms statistically extracts the displacements by correlating fitting calibrating observation system elements. The algorithms correlate the values of neighboring observation-system pixels and locally determine the displacement down to 1/100th the pixel pitch. The ultimate accuracy depends on pixel signal-to-noise ratios and the image content.

2. Phase correlation algorithm
The phase correlation method is a well-known technique in estimating displacements, or estimating general transformations such as affined transformations [4]. Phase correlation method has also been extended into sub-pixel accuracy with varying levels of success [5, 6]. It has also been used to estimate rotations [7]. Phase correlation method can be made particularly robust in the presence of band-limited noises and distortions. Since the phase difference for every frequency contributes to the resulting displacement estimate, the location
of the peak, which represents the displacement, will not change if there is noise, which is limited to a narrow bandwidth [4].

Phase correlation-based algorithms take advantage of the shift property of Fourier transform. The shift (transformation, delay, etc.) property in the 2-D case is defined as follows,

\[ f(x - x_0, y - y_0) = F(u, v) \cdot \exp(-j2\pi (ux_0 + vy_0)) \]  \hspace{1cm} (1)

Where \( x_0 \) and \( y_0 \) are shifts (displacements) in the and directions, respectively, and \( F(u, v) \) denotes the Fourier transform of a 2-D function (image) \( f(x, y) \).

The shift operator imposes a constant group shift, thereby exhibiting linear phase, as evidenced by the equality of (2) which results directly from (1). Equation (2) shows directly that the operation of a shift operator results in a linear phase shift, with phase shifts varying linearly with frequency.

\[ H(u, v) = \frac{F_{\text{output}}(u, v)}{F_{\text{input}}(u, v)} = \exp(-j2\pi (ux_0 + vy_0)) \]  \hspace{1cm} (2)

Phase correlation-based algorithms uses the cross-power spectrum of the two images to derive the phase difference:

\[ \frac{F_{\text{output}}(u, v) \cdot F^*_\text{input}(u, v)}{F_{\text{input}}(u, v) \cdot F^*_\text{input}(u, v)} = \exp(-j2\pi (ux_0 + vy_0)) \]  \hspace{1cm} (3)

Where \( F^* \) is the complex conjugate of \( F \). Then, the algorithms rely on the inverse Fourier transform to locate the peak of the correlation. The inverse Fourier transform generates a delta function \( p \) with an offset corresponding to the displacement, which is defined as follows,

\[ p = F^{-1}(\exp(-j2\pi (ux_0 + vy_0))) \]  \hspace{1cm} (4)

The method is generally very efficient when only pixel-level displacement accuracy is required.

3. Correlation fitting calibration algorithm

We derive a completely analytical algorithm for displacement estimation that models an arbitrary correlation surface as the general second-order two dimensional Taylor-series expansion \( f(x, y) = a_{00} + a_{01}x + a_{10}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 \). We formulate the general linear least-squares problem by comparing the data stored in the 3x3 (or 5x5) correlation grid with the expected values computed using the second-order six-parameter model. We then compute the optimum values of these six parameters by minimizing analytically the corresponding sum of the squared errors, obtaining a solution set consisting of simple linear combinations of the correlation data values. As shown in Fig. 1 below.

![Fig. 1. Images of correlation function.](image-url)
The surface is curve-fitted to a general second-order surface, yielding the high-resolution curve. Since there are six coefficients in the second-order fitting function \( a_{00} a_{10} a_{01} a_{20} a_{02} \).

We propose to use the well-known method of least-squares fitting, which is identical to the method of maximum likelihood estimation [8] of the fitted parameters if the instrumental and statistical errors in the correlation data are both independent and follow the normal distribution.

Further, we seek a minimum of \( f(x, y) \) by asserting that the gradient with respect to the Cartesian coordinates \( \{x, y\} \) equals to zero at \( \{x_0, y_0\} \):

\[
\frac{\partial}{\partial x} f(x, y) \bigg|_{(x,y) \rightarrow (x_0,y_0)} = 0 \\
\frac{\partial}{\partial y} f(x, y) \bigg|_{(x,y) \rightarrow (x_0,y_0)} = 0
\]

Solving these two equations yield \( \{x_0, y_0\} \),

\[
\begin{cases}
x_0 = \frac{a_{01}a_{11} - 2a_{10}a_{02}}{4a_{20}a_{02} - a_{11}^2} \\
y_0 = \frac{a_{10}a_{01} - 2a_{01}a_{20}}{4a_{20}a_{02} - a_{11}^2}
\end{cases}
(6)
\]

This is the maximum likelihood displacement estimation.

In the variation we propose, the denominator \( 4a_{20}a_{02} - a_{11}^2 \) is determined immediately after the reference frame is first acquired, and then held fixed for subsequent comparison frames. So, the denominator \( 4a_{20}a_{02} - a_{11}^2 \) is held constant, and \( \{x_0, y_0\} \) depends linearly on the 9 correlation numbers through \( a_{10} \) and \( a_{01} \). Therefore, we could expect that the navigation errors arising from the use of the algorithm would be linear in the computed values of \( \{x_0, y_0\} \). Consequently, the correction functions which are linear in \( \{x_0, y_0\} \) can be found and applied after the displacement has already been computed using algorithm. By definition, these linear functions have the form

\[
\Delta x(x_0, y_0) = p_{00} + p_{10}x_0 + p_{01}y_0
\]

\[
\Delta y(x_0, y_0) = q_{00} + q_{10}x_0 + q_{01}y_0
\]

where \( p \) and \( q \) are fitting coefficients and can easily be fitted to the displacement estimation errors. Equation (9) below shows how the correction terms may be applied to computed displacement \( \{x_0, y_0\} \).

\[
x_{i,j}^* = x_{i,j} - \Delta x(x_{i,j}, y_{i,j})
\]

\[
y_{i,j}^* = y_{i,j} - \Delta y(x_{i,j}, y_{i,j})
\]

Significantly reductions in both navigation RMS errors and maximum errors were observed after applying the algorithm to pre-calibrated targets.

4. Analysis and discussion of the experimental result

The motion images of MEMS resonator at different driving frequencies and different motion phases needs to obtain before applying Phase correlation algorithms. The images are got using stroboscopic imaging technique. The motion frequency for MEMS is very high (10K~500KHz). At the condition the images of motion MEMS got from CCD are blur (see Fig. 2). The principle of stroboscopic imaging is using strobe light to illuminate motion MEMS device the strobe light is frozen at one position when the motion frequency of MEMS equal to strobe frequency. At the position through exposure several times using CCD the clear
motion image of MEMS device is got. Figure 3 shows the clear image of MEMS at one of the driving frequency and motion phase through stroboscopic imaging method.

The MEMS device are mounted on the moving part of a nanostage. The sensor used for this research is a monochrome CCD sensor of 1534×1024 pixels with a 9.0 μm pixel size. The optic lens used is 50×/0.80 high-numerical-aperture Olympus microscope objective lenses, which provide a calibrated pixel size on the target of 182.05 nm respectively. The illumination is broad white light coming in through bright-field (normal) direction. The moving part of the stage is moved against the stationary part of the stage in a commanded step size of 5 nm through a (0,+5,+10,+15,+10,+5, 0) pattern over a period of about one minute. There are more than 400 frames acquired in the experiment and 50 frames for each position are chosen to conduct algorithm studies.

Figure 4 shows raw displacement results computed from 400 image frames, plotted at the four commanded stage positions, which are 0 nm, 5 nm, 10 nm and 15 nm. The results have a very small variance although displacements have an offset from the commanded stage position. The offset, which is within the over-all position uncertainty of the whole system, may be due to errors in the commanded movement or from sensing. From the histogram in Fig. 5, it is obvious that the algorithm with such an optical system can potentially detect displacements much smaller than the 5 nm step size used in these experiments. The measured standard deviation 3σ is less than 1nm, which may include variations due to the commanded motion, motion hysterisis, or low frequency drifts. Thus, it is possible to achieve displacement detection of better than a 1 nm or 1/100th of a pixel size.

Applied technique of correlation fitting calibration algorithm, motion amplitudes corresponding to the nine phases are obtained. Figure 6 shows the phase-amplitude curve of MEMS resonator at 22KHz. From Fig. 6 the motion track of MEMS resonator in one cycle is clear.
Experiment results indicate that phase correlation algorithms can describe instant motion status for moving MEMS device. It can plot cycle motion trace of MEMS device.

5. Conclusion
Phase correlation method is the most common motion estimation method in present. It meets real-time and accurate at the same time for many applications. In this paper, details of correlation fitting calibration algorithms were presented. Using stroboscopic imaging technique the clear images of motion MEMS device in different driving frequencies and different motion phases are obtained. And we can extract in-plane motion characteristics of MEMS devices that give important reference to design of MEMS. Experimental results obtained from the algorithms demonstrated that the accuracy and precision could reach 5nm or beyond using conventional diffraction-limited optical microscopy.

References